Chapter Six: X-Rays

6.1 Discovery of X-rays

In late 1895, a German physicist, W. C. Roentgen was working with a cathode ray tube in his laboratory. He was working with tubes similar to our fluorescent light bulbs. He evacuated the tube of all air, filled it with a special gas, and passed a high electric voltage through it. When he did this, the tube would produce a fluorescent glow. Roentgen shielded the tube with heavy black paper, and found that a green colored fluorescent light could be seen coming from a screen setting a few feet away from the tube. He realized that he had produced a previously unknown "invisible light," or ray, that was being emitted from the tube; a ray that was capable of passing through the heavy paper covering the tube. Through additional experiments, he also found that the new ray would pass through most substances casting shadows of solid objects on pieces of film. He named the new ray X-ray, because in mathematics "X" is used to indicated the unknown quantity.

In his discovery Roentgen found that the X-ray would pass through the tissue of humans leaving the bones and metals visible. One of Roentgen's first experiments late in 1895 was a film of his wife Bertha's hand with a ring on her finger. The news of Roentgen's discovery spread quickly throughout the world. Scientists everywhere could duplicate his experiment because the cathode tube was very well known during this period. In early 1896, X-rays were being utilized clinically in the United States for such things as bone fractures and gun shot wounds.

6.2 Production of X-rays

An X-ray tube is a vacuum tube designed to produce X-ray photons. The first X-ray tube was invented by Sir William Crookes. The Crookes tube is also called a discharge tube or cold cathode tube. A schematic x-ray tube is shown below.

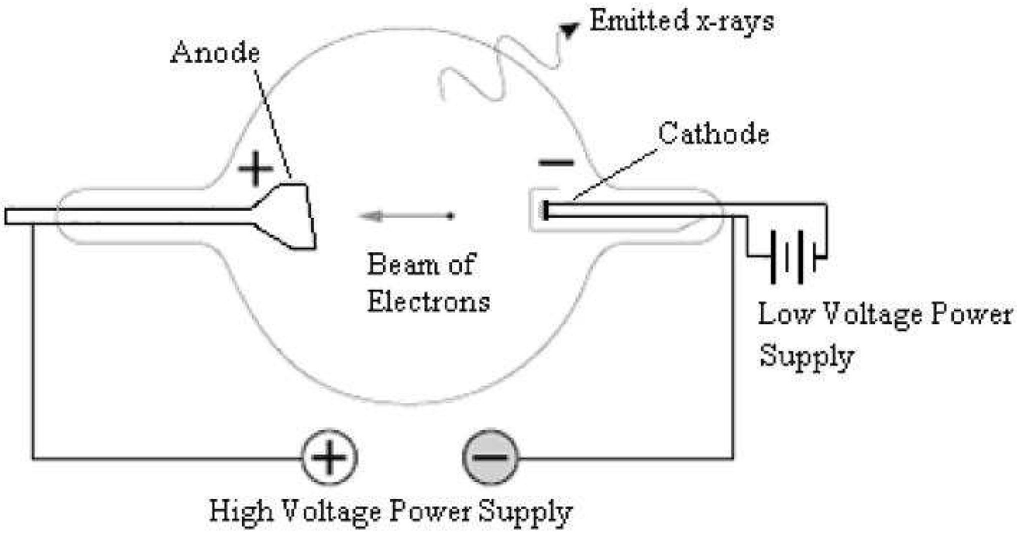


Fig.6.1: A Schematic Diagram of an X-Ray Tube

The glass tube is evacuated to a pressure of air, of about 100 pascals, recall that atmospheric pressure is 10^6 pascals. The anode is a thick metallic target; it is so made in order to quickly dissipate thermal energy that results from bombardment with the cathode rays. A high voltage, between 30 to 150 kV, is applied between the electrodes; this induces an ionization of the residual air, and thus a beam of electrons from the cathode to the anode ensues. When these electrons hit the target, they are slowed down, producing the X-rays. The X-ray photon-generating effect is generally called the Bremsstrahlung effect,

a contraction of the German "brems" for braking, and "strahlung" for radiation.

The radiation energy from an X-ray tube consists of discrete energies constituting a line spectrum and a continuous spectrum providing the background to the line spectrum.

6.3 Properties of X-rays

- **X-rays travel in straight lines.**
- **X-rays cannot be deflected by electric field or magnetic field.**
- **X-rays have a high penetrating power.**
- **Photographic film is blackened by X-rays.**
- **Fluorescent materials glow when X-rays are directed at them.**
- **Photoelectric emission can be produced by X-rays.**
- **Ionization of a gas results when an X-ray beam is passed through it.**

6.4 Continuous Spectrum

When the accelerated electrons (cathode rays) strike the metal target, they collide with electrons in the target. In such a collision part of the momentum of the incident electron is transferred to the atom of the target material, thereby loosing some of its kinetic energy, ΔK. This interaction gives rise to heating of the target. The projectile electron may avoid the orbital electrons of the target element but may come sufficiently close to the nucleus of the atom and come under its influence. The projectile electron we are tracking is now beyond the K-shell and is well within the influence of the nucleus. The electron is now under the influence of two forces, namely the

attractive Coulomb force and a much stronger nuclear force. The effect of both forces on the electron is to slow it down or decelerate it. The electron leaves the region of sphere of influence of the nucleus with a reduced kinetic energy and flies off in a different direction, because the vector velocity has changed. The loss in kinetic energy reappears as an x-ray photon, as illustrated in Fig.6.2. During deceleration, the electron radiates an X-ray photon of energy $h\nu = \Delta K = K_i - K_f$ **. The energy lost by incident electrons is not the same for all electrons and so the x-ray photons emitted are not of the same wavelength. This process of X-ray photon emission through deceleration is called *Bremsstrahlung* and the resulting spectrum is continuous but with a sharp cut-off wavelength. The minimum wavelength corresponds to an incident electron losing all of its energy in a single collision and radiating it away as a single photon.**

If K is the kinetic energy of the incident electron, then

$$K = h\nu = \frac{hc}{\lambda_{\min}}$$

The cut off wavelength depends solely on the accelerating voltage.

$$h\nu_{\max} = \frac{hc}{\lambda_{\min}} = eV$$

, where *V* is the accelerating voltage.

$$\nu_{\max} = \frac{eV}{h}$$

$$\lambda_{\min} = \frac{hc}{eV}$$

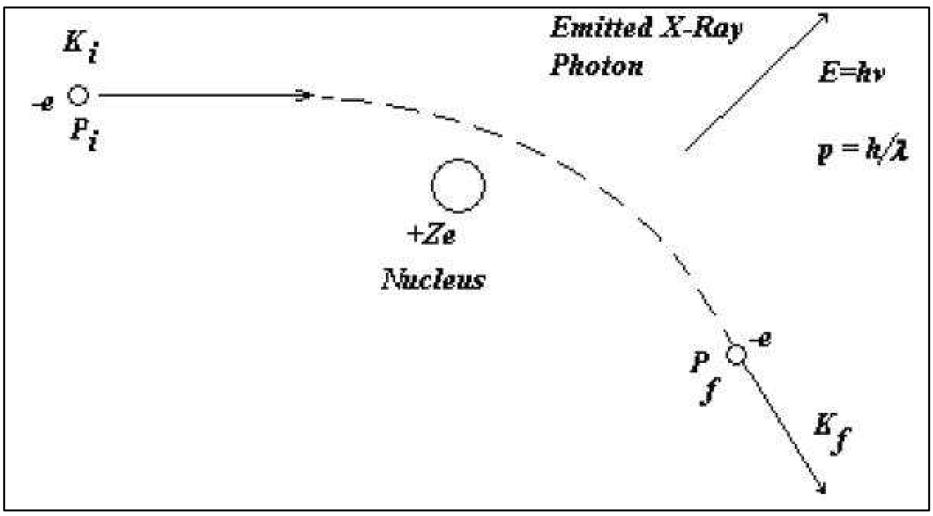


Fig. 6.2: Deceleration of an Electron by a Positively Charged Nucleus

6.5 Characteristic X-Ray Spectrum

Because of the large accelerating voltage, the incident electrons can

(i) Excite electrons in the atoms of the target.

(ii) Eject tightly bound electrons from the cores of the atoms.

Excitation of electrons will give rise to emission of photons in the optical region of the electromagnetic spectrum. However when core electrons are ejected, the subsequent filling of vacant states gives rise to emitted radiation in the x-ray region of the electromagnetic spectrum. The core electrons could be from the K-, L- or M- shell.

If K-shell (n=1) electrons are removed, electrons from higher energy states falling into the vacant K-shell states, produce a series of lines denoted as K_α, K_β ,... as shown Fig.6.3.

Transitions to the L shell result in the L series and those to the M shell give rise to the M series, and so on.

Since orbital electrons have definite energy levels, the emitted X-ray photons also have well defined energies. The emission spectrum has sharp lines characteristic of the target element.

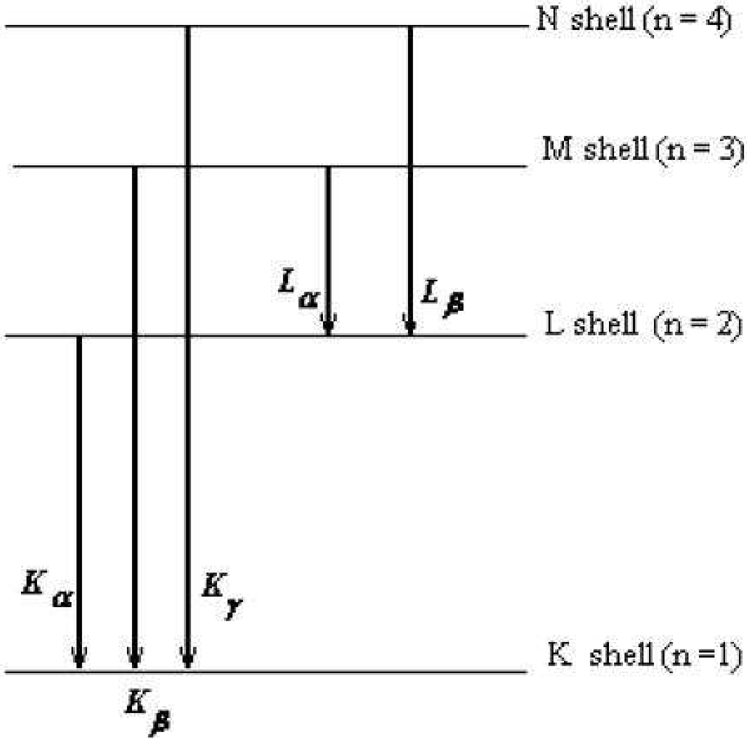
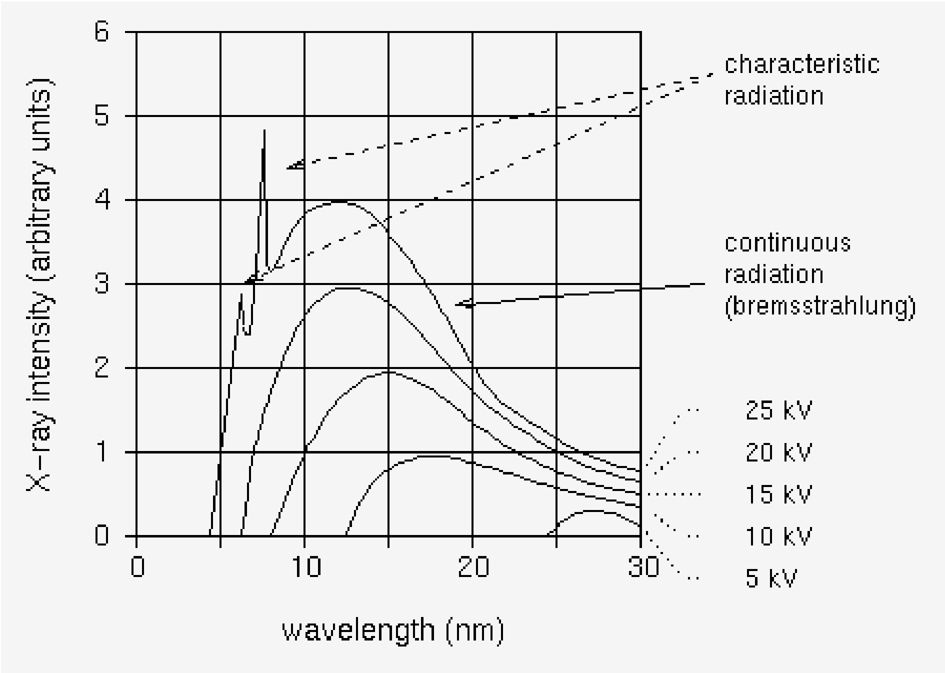


Fig.6.3: X–Ray Transitions

Not all transitions are allowed. Only those transitions which fulfill the following selection rule are allowed: $\Delta\lambda = \pm 1$.



- **The graph shows the following features.**
  - **A continuous background of X-radiation in which the intensity varies smoothly with wavelength. The background intensity reaches a maximum value as the wavelength increases, and then the intensity falls at greater wavelengths.**
  - **Minimum wavelength which depends on the tube voltage. The higher the voltage the smaller the value of the minimum wavelength.**
  - **Sharp peaks of intensity occur at wavelengths unaffected by change of tube voltage.**

6.6 X-Ray Diffraction

A plane of atoms in a crystal, also called a Bragg plane, reflects X-ray radiation in exactly the same manner that light is reflected from a plane mirror, as shown in Fig.6.4.

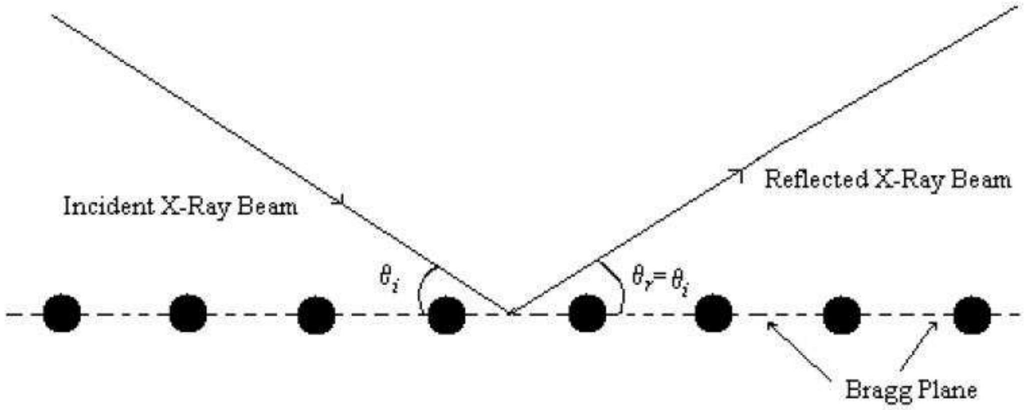


Fig. 6.4: X-Ray Reflection from a Bragg Plane

Reflection from successive planes can interfere constructively if the path difference between two rays is equal to an integral number of wavelengths. This statement is called Bragg's law.

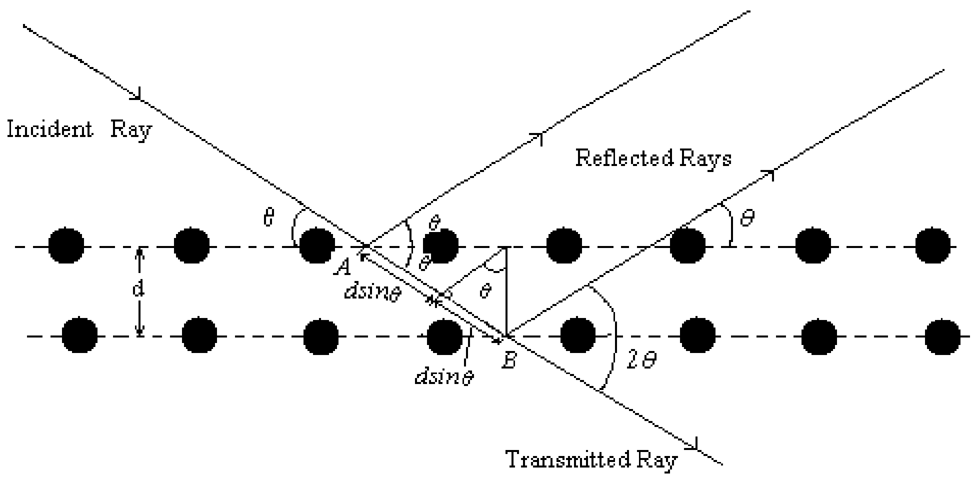


Fig. 6.5: Diffraction of X-Rays from Atomic Planes

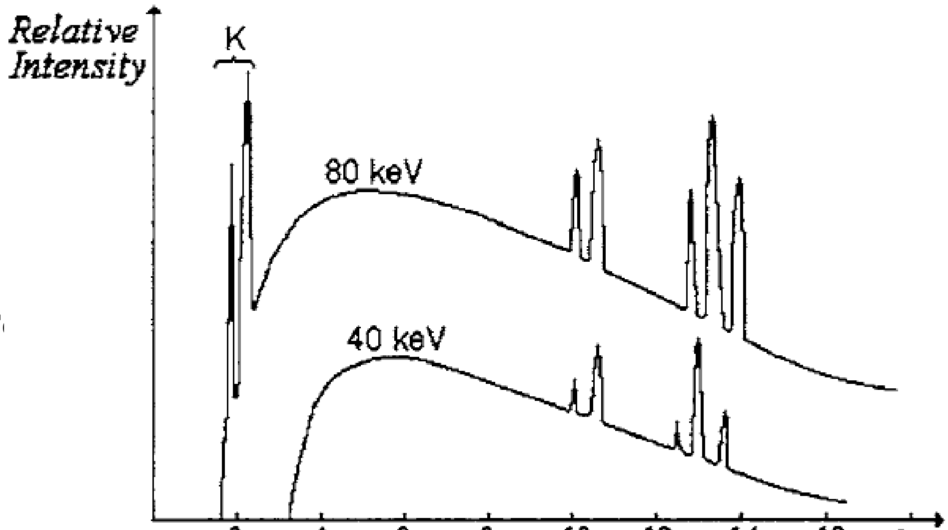
From Fig. 4.8, AB = 2dsinθ so that by Bragg's law, we have

$$2d \sin\theta = n\lambda$$

Where in practice, it is normal to assume first order diffraction so that n = 1. A given set of atomic planes gives rise a reflection at one angle, seen as a spot or a ring in a diffraction pattern also called *a diffractogram*.

6.7 Moseley's Experiment

The high intensity penetrating radiation emitted by X-ray tubes, characteristic of the metal from which the target anode is made, was first discovered by Barkla. He found that when the tubes were operated at higher potentials, series of high intensity peaks, each of a specific wavelength, were superimposed on the spectrum of the continuous bremstrahlung radiation (Fig. 6.5).



The phenomenon is analogous to the atomic line spectra Seen in the visual region of the electromagnetic spectrum. Changing the metal or element from which the target anode in the X-ray tube is made alters the wavelengths at which the high intensity peaks occur. The most penetrating series in an element's characteristic X-ray spectrum is called the K series; the second is called the L series; the third the M series and so on.

Moseley carried out a systematic examination of the characteristic radiation of as many elements as possible. He examined the X-ray spectra of the 38 elements from aluminum (Al) to gold (Au). As regards 15 of these elements, he studied just the K series; regarding another 17, just the L series; as to the remaining 6 elements, both series. He recorded the spectra on photographic plates.

Moseley discovered the following simple empirical relationship, illustrated in (Fig 6.6), between the frequencies, (ν) of the lines in each series and the ordinal number, N, of the element's position in the periodic table (starting from hydrogen):

$$\sqrt{\nu} = a(N - \sigma)$$

where σ called the screening constant.

Moseley formed the opinion that some physical attribute of the atom must increase by (a) regular fixed amount, from one element to the next, rising through the periodic table. He postulated that this could only be the atom's nuclear charge.

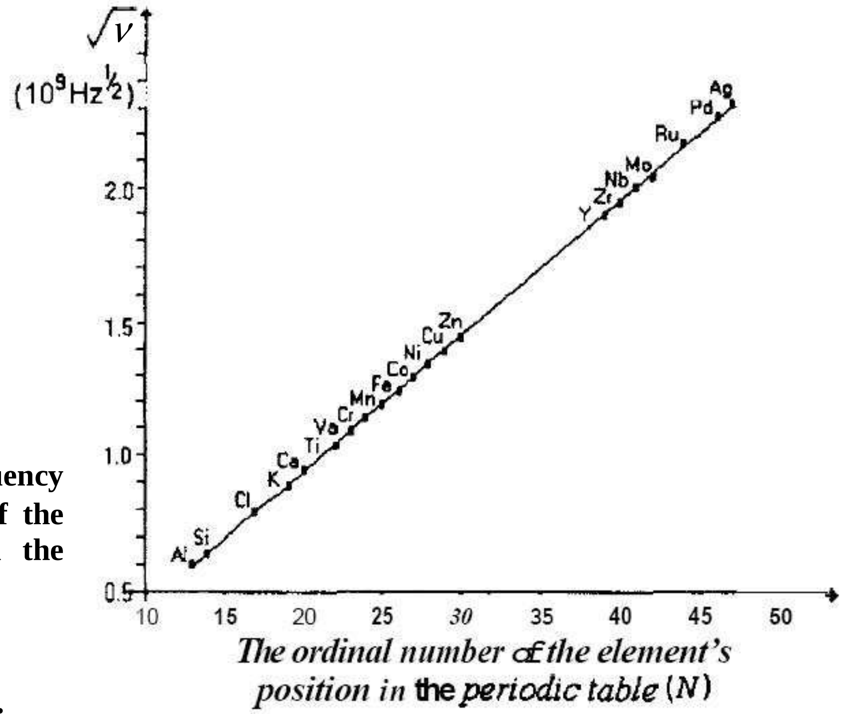


Fig. 6.6: The square root, $\sqrt{\nu}$ of the frequency of an element's K, line as a function of the ordinal number, N, of its position in the periodic table.

According to this hypothesis, the number N, that is the element's ordinal position in the periodic table, is equal to the number of natural units of positive electricity carried by the nuclei of the element, i.e., N=Z . The number Z is now called the *atomic number* of the element; it is equal to the number of protons in the element's nuclei.

Prior to Moseley's investigation, the elements were arranged in the periodic table in the ascending order of their atomic weights and on the basis of their chemical properties. As a result of Moseley's researches, which provided the first direct means of determining an element's atomic number, inaccuracies in the periodic table were discovered and corrected. For example, the positions of the transition metals cobalt (Z = 27) and nickel (Z = 28), that had been previously determined by the ascending order of their atomic weights, Ni = 58.71 and Co = 58.93, were switched. Similarly, empty positions

were revealed in the table, corresponding to the yet undiscovered elements of atomic number 43, 61, 72 and 75.

The origin of the characteristic X-ray radiation is readily explained by the Bohr model of the atom. Let us assume that the electrons orbiting the nucleus in many electron atoms are arranged in shells, each electron having its specific slot in a shell. We will designate the innermost shell as the K shell it corresponds to the principal quantum number (n=1). The second shell is designated the L shell; it corresponds to the principal quantum number (n=2). The third shell is designated the M-shell; it corresponds to the principal quantum number (n=3), and so on. According to this model, all the electrons in a particular shell have the same energy and the closer a shell is to the nucleus the greater the energy binding its electrons to the atom. The electrons in the K shell are all in the energy level E_1, those in the L shell in the level E_2, and so on (see Fig. 6.7).

When an electron, with sufficiently high energy, strikes an atom in the X-ray tube anode, it ejects an electron from one of the atom's inner shells, say the K shell. This leaves an empty slot or hole in this shell. One of the electrons from an outer shell, corresponding to an energy level E_n can 'fall' into this hole, releasing an amount of energy, E_n-E_1, equal to the difference between the energy levels of the two shells; this energy is released as an X-ray photon of frequency:

$$\nu = \frac{E_n - E_1}{h}$$

On the basis of this explanation, the K series results from the filling of holes in the K shell (n=1), with the K, line corresponding to the hole being filled by an

electron falling from the L shell; the K_β line to the electron falling from the M shell, and so on.

Similarly, the L series results from the filling of holes in the L shell (n = 2). The more electrons there are in an atom, the more series there will be in its X-ray spectrum and the more lines there will be in each series. An atom's electronic energy levels depend on the interaction between its electrons and its nuclear charge, +Ze, and so, since the atoms of each element carry a characteristic nuclear charge, each element exhibits a characteristic X-ray spectrum.

To a close approximation, the frequencies, ν_{K_α} of the K-line, the most intense line in an element's K series -are given by the formula:

$$\nu_{K_\alpha} = cR_\infty (Z-1)^2 \left(\frac{1}{1^2} - \frac{1}{2^2} \right) \qquad (6.1)$$

It is as though after the removal of an electron from the K shell, the electron in the L shell sees a nuclear charge of (Z - 1). The frequencies, ν_{L_α} of the L_α line-the most intense line in the element's L-series are given by the formula:

$$\nu_{L_\alpha} = cR_\infty (Z-7.4)^2 \left(\frac{1}{2^2} - \frac{1}{3^2} \right) \qquad (6.2)$$

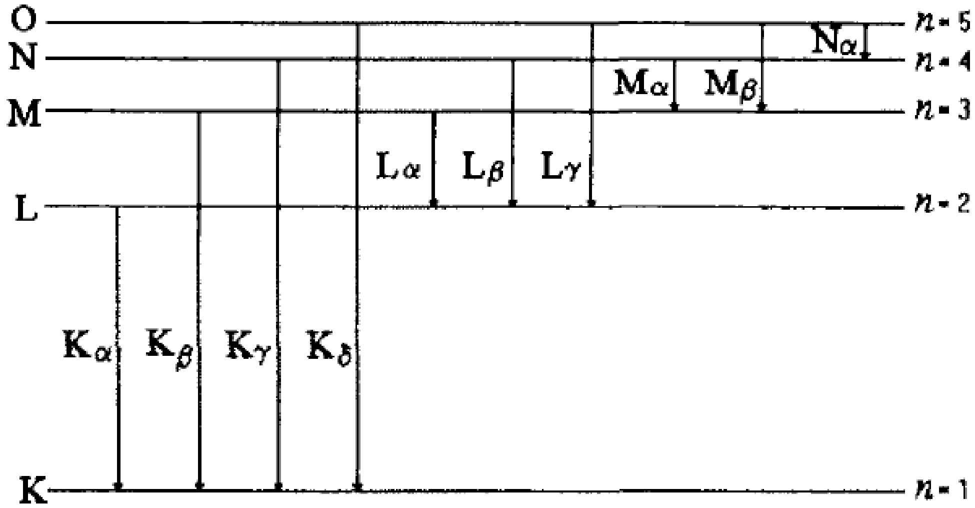


Fig. 6.7: The transitions that produce the characteristics X-rays radiations. A 'hole' in an inner shell of the atom's electron structure is filled by an electron that 'falls' from a higher energy level; the difference in energy appears as high energy photon.

On account of the relatively large nuclear charge of the anode metals (Z > 30), the motion of their innermost electrons is virtually unaffected by the outermost electrons.

The innermost electrons interact almost exclusively with the nucleus, just like the electron in the hydrogen atom. Consequently, as in Bohr's elucidation of the hydrogen spectrum, the denominators of the fractions in equations (6.1) and (6.2) denote the principal quantum numbers of the energy levels between which the electronic transitions have taken place. The screening constant, σ, is greater in equation (6.2) because the electrons in the L shell are shielded from the nuclear charge by the innermost electrons in the K shell.

Accurate measurements show, that each principle line in the characteristic X-ray spectrum is in fact composed of a fine structure of very close discrete lines. For example, the K-line comprises two lines; it is a doublet. It follows, that when an electron falls from the L shell to the K shell, there are two possible values for the difference, E_2 -E_1, between the energy levels of the shells. This indicates that there are sub-levels within the principle energy levels of the atom.

Example: The Characteristic X-ray Spectrum of Copper (Cu)

Estimate,

(i) the frequency of the K_β line in the spectrum of the X-rays emitted from an X-ray tube with a copper anode;

(ii) the minimum potential at which the tube must be operated for this line to appear.

Calculation: (i) The K_β line corresponds to a transition from the M shell to the K shell. It is apparent from (Fig. 6.7) that

$$\nu_{K_\beta} = \nu_{K_\alpha} + \nu_{L_\alpha}$$

The atomic number of copper (Cu) is 29 and so from equation (6.1)

$$\nu_{K_\alpha} = 3.291\times10^{15}(29-1)^2\left(\frac{1}{1^2}-\frac{1}{2^2}\right) = 1.935\times10^{18}\ \text{Hz}$$

and from equation (6.2)

$$\nu_{L_\alpha} = 3.291\times10^{15}(29-7.4)^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right) = 0.213\times10^{18}\ \text{Hz}$$

which gives for the frequency of the K_β line:

$$\nu_{K_\beta} = (1.935+0.213)\times10^{18} = 2.148\times10^{18}\ \text{Hz}$$

(ii) For any of the K lines to appear in the X-ray spectrum, an electron must be dislodged from the K shell. The binding energy of these electrons can be estimated by assuming that the conditions under which they orbit the nucleus are comparable to those of the electron in a hydrogen-type atom, i.e., that they interact almost exclusively with the nucleus. Substituting Z=29 in the equation

$E_n = -13.6\,\dfrac{Z^2}{n^2}\ eV$ **with n=1 (because we are in the K=shell).**

$$E_1 = -13.6\times(29)^2 = 11{,}440\quad eV$$

Thus, in order to dislodge a K shell electron in copper, we might estimate that they must be struck by an electron that has been accelerated through a potential of 1 1,440 V at least.