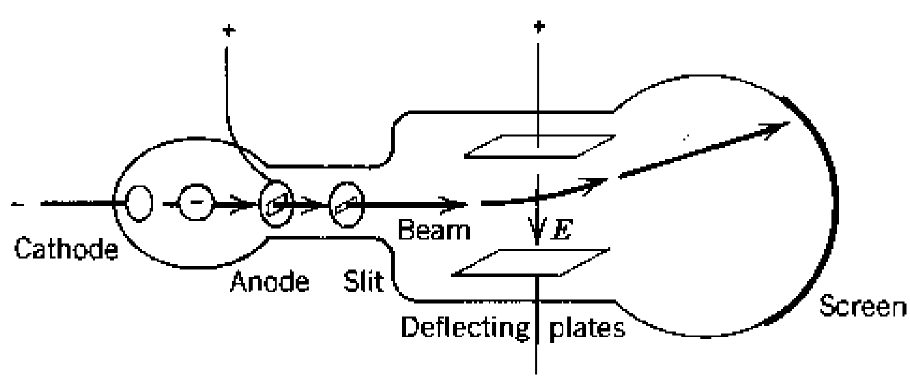
1. Historical perspective

In the years before Bohr formulated his theory of the atom, based upon the principles of quantum physics, some steps had been made on the understanding of the atomic structure. We list here some important contributions:

- **Mendeleev had developed a concept for arranging the known chemical elements based on their mass. Order was given in terms of increasing mass, while the elements were further arranged according to the ordering principle of chemical behaviour. The columns in the table of the elements relate to chemical valence. In 1869 the Periodic Table was not yet complete.**
- **Avogadro had conceived the idea that gasses consist of discrete particles and had established the law that equal volumes of gas at equal pressure and temperature contain the *same* number of such particles, although the actual number was not yet determined. Avogadro's hypothesis was not accepted by many physicists for a long time.**
- **It was realized for some time that electrostatic charges were important for the building blocks of matter. This followed from Faradays experiments on electrolysis , from which it was deduced that ions move in a liquid as charged particles, and from the experiments on radioactivity in which electrically charged particles were emitted.**
- **Thomson's experiments on cathode rays were important for the determination of some properties of the constituents of matter. The charged particles, emitted from a cathode, were deflected in a combination of crossed static electric and magnetic fields, and detected on the phosphorent screen (see Fig). Hence impinging charged particles could be made visible by the light emitted by the screen.**

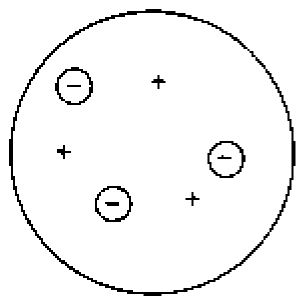


- **After the ratio (e/m) was determined Millikan performed his famous [oil-drop experiment] from which the two values of e and m could be unravelled.**

Chapter one: Atomic Models

1.1 Thomson's model

Based on these concepts Thomson developed a model for the atom consisting of the electrons as negatively charged particles of low mass and some substance that should carry positive charge and nearly all the mass within the atom. Since the elements were arranged according to their mass number A, the atoms were thought to consist of A positive particles and A electrons in a structure as shown below. Note that the atomic number Z does not play a role yet.

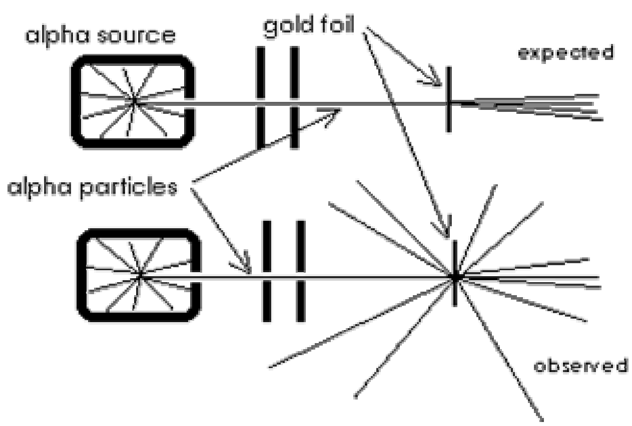


1.2 Rutherford scattering

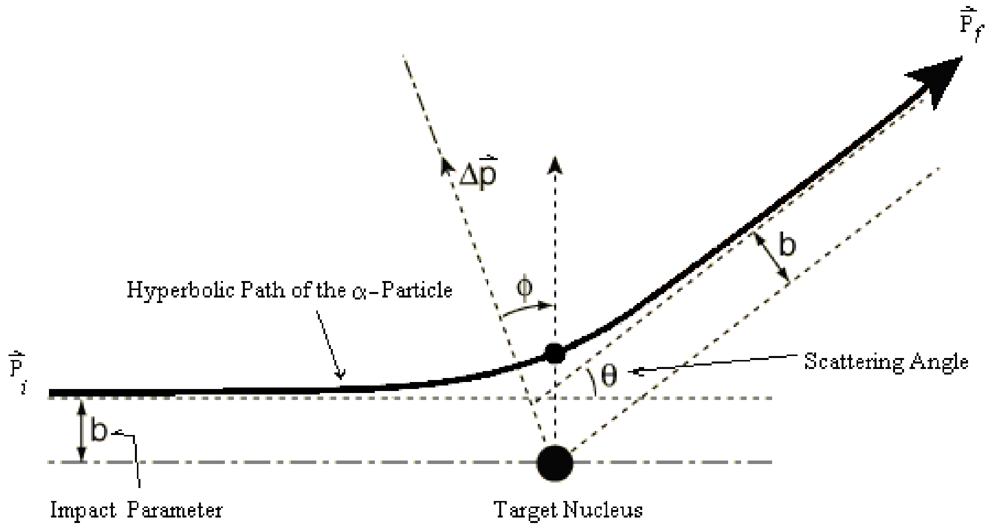
Rutherford scattering is a phenomenon that was explained by Ernest Rutherford in 1911, and led to the development of the orbital theory of the atom. It is now exploited by the materials analytical technique Rutherford backscattering. Rutherford scattering is also sometimes referred to as Coulomb scattering because it relies on static electric (Coulomb) forces.

Highlights of Rutherford's Experiment:

- **A beam of α particles were aimed at a thin gold foil.**
- **Most of the particles passed through without deflection.**
- **Others were deflected by various angles.**
- **Some were backscattered.**



From these results Rutherford concluded that the majority of the mass was concentrated in a minute, positively charged region (the nucleus) surrounded by electrons. When a (positive) alpha particle approached sufficiently close to the nucleus, it was repelled strongly enough to rebound at high angles. The small size of the nucleus explained the small number of alpha particles that were repelled in this way. Rutherford showed,using the method below, that the size of the nucleus was less than about 10^{-14} m .



The relationship between the scattering angle θ, the initial kinetic energy $K = \frac{1}{2}mv_0^2$ **and the impact parameter b is given by,**

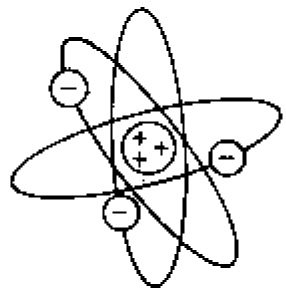
$$b = \frac{zZ}{2K}\frac{e^2}{4\pi\varepsilon_0}\cot\left(\frac{\theta}{2}\right),$$ **where z =2 for α-particle and Z = 79 for gold.**

1.3 Rutherford's model of the atom

Sir Ernest Rutherford proposed a model of the atom based on the results of alpha particle scattering that the atom consisted mainly of empty space with a tiny, positively charged nucleus, containing most of the mass of the atom, surrounded by negative electrons in orbit around the nucleus like planets orbiting the Sun.

According to Maxwell's electromagnetic theory, a charged particle in circular motion radiates energy and so an electron in a Rutherford's atom should continuously lose energy as it moves in a planetary orbit and eventually should spiral down to the nucleus at the centre of the atom, which does not happen. Rutherford's model though a much improved picture of the atom, but could not explain stability of the atom.

Furthermore, according to classical physics, the energy emitted by an electron as it spirals down to the nucleus should have all frequencies, in other words the emitted spectrum should be continuous which is not the case. The emitted spectrum consist of lines in a dark background. Thus, Rutherford's model could not explain the observed line spectra of elements.



There are however a number of shortcomings to this planetary model of an atom bound by classical electromagnetic forces:

- **the problem of stability with accelerated electrons in orbit loosing energy.**
- **the model gives no indication of the size of the atom.**
- **the model gives no explanation for the characteristic spectroscopy of atoms.**

1.4 The Bohr model of the atom

Niels Bohr proposed an atomic model that would explain the discrepancies between the observed line spectra emitted by elements and the spectra predicted by the Rutherford's atomic model.

Bohr proposed the following postulates:

1. **An electron in an atom moves in a circular orbit about the nucleus under the influence of the Coulomb force between the electron and the nucleus.**
2. **An electron moves in an orbit for which its orbital angular momentum L is an integral multiple of $\hbar$ ($L = n\hbar$).**
3. **An electron moving in an allowed orbit does not radiate electromagnetic energy. Thus, its total energy E remains constant.**
4. **Electromagnetic radiation is emitted if an electron, initially moving in an orbit of total energy E_i, discontinuously changes its motion so that it moves in an orbit of total energy E_f. The frequency ν of the emitted radiation is equal to the quantity $\left(\nu = \frac{E_f - E_i}{\hbar}\right)$.**

1.5 Bohr-Sommerfield model

Bohr, in his semiclassical analysis, had only allowed for circular orbits in his model. Later the model was extended by Sommerfeld also allowing for elliptical orbits. This version, based on the same ad hoc quantization condition for the angular momentum is referred to as the Bohr-Sommerfeld model. Sommerfeld already postulated the azimuthal quantum number, in addition to the principle quantum number n defined by Bohr.

Home work:

Q.1 In the lectures given to you, there seems to have been disagreement between Niels Bohr and Rutherford. What was the disagreement about and how was it resolved?

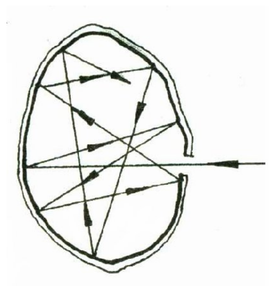
Q.2 Did Rutherford's model explain (a) the stability of atoms? (b) why atoms emit discrete wavelengths?

Q.3 A 6.0 MeV α-particle is scattered at 40° by a gold nucleus. What is the corresponding impact parameter?

Chapter Two: Quantum Theory

2.1 Black body

A blackbody is defined as the body which can absorb all energies that fall on it. It is something like a black hole. No lights or material can get away from it as long as it is trapped. A large cavity with a small hole on its wall can be taken as a blackbody.



2.2 Black body radiation

Any radiation that enters the hole is absorbed in the interior of the cavity, and the radiation emitted from the hole is called blackbody radiation.

2.2.1 Stefan-Boltzmann Law :

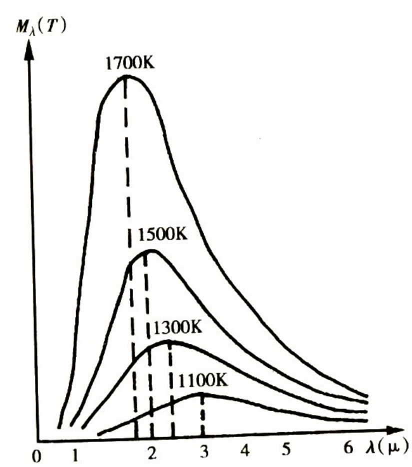
It is found that the radiation energy is proportional to the fourth power of the associated temperature.

$$M_\lambda(T) = \sigma T^4$$

Where $M_\lambda(T)$ **is the area under the curve**

σ **is called stefan's constant**

T is the absolute temprature in Kelvin



2.2.2 Wien's displacement law:

The peak of the curve shifts towards longer wavelength as the temperature falls and it satisfies

$$\lambda_{\max} T = b$$

Where *b* is called the Wien's constant

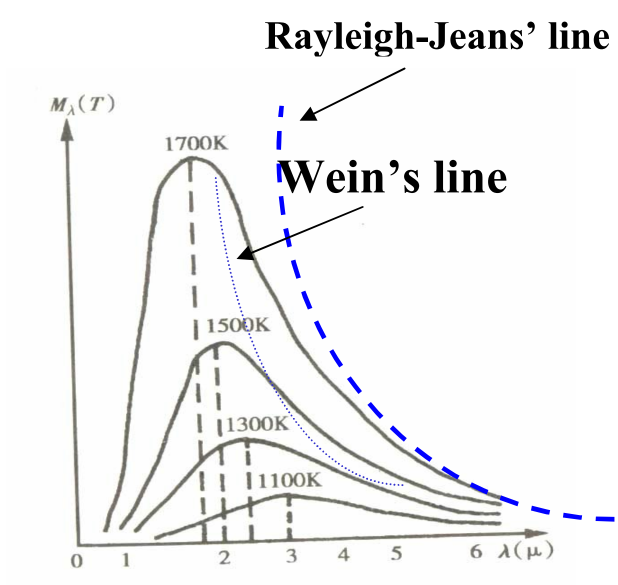
This law is quite useful for measuring the temperature of a blackbody with a very high temperature.

2.3 Rayleigh-Jeans Law

In 1890, Rayleigh and Jeans obtained a formula using the classical electromagnetic (Maxwell) theory and the classical equipartition theorem of energy in thermodynamics. The formula is given by;

$$M_\lambda(T) = C_1 \lambda^{-4} T$$ **where C_1 is a constant**

Rayleigh-Jeans formula was correct for very long wavelength in the far infrared but hopelessly wrong in the visible light and ultraviolet region. Maxwell's electromagnetic theory and thermodynamics are known as correct theory. The failure in explaining blackbody radiation puzzled physicists! It was regarded as ultraviolet Catastrophe (disaster).

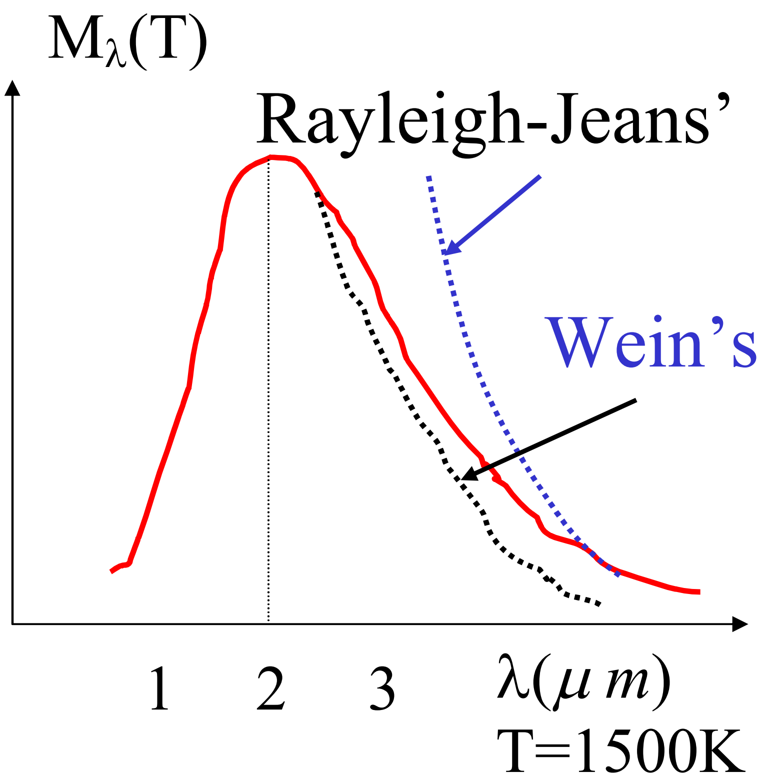


2.4 Wein's formula:

Later on in 1896, Wein derived another important formula using thermodynamics.

$$M_{\lambda}(T) = C_2\, \lambda^{-5}\, e^{-\frac{C_3}{\lambda T}}$$

Unfortunately, this formula is only valid in the region of short wavelengths.



2.5 Planck's Magic formula

In 1900, after studying the above two formulas carefully, Planck proposed an empirical formula

$$M_\lambda(T) = 2\pi \hbar c^2 \lambda^{-5} \frac{1}{e^{\frac{hc}{\lambda kT}} - 1}$$

Where c is the speed of light, *k* is Boltzmann's constant, h is Planck's constant and e is the base of natural logarithm.

It is surprising that the experience formula can describe the curve of blackbody radiation exactly for all wavelengths.

- **Other unbelievable deductions:**

(1) For very large wavelength, the Rayleigh-Jeans formula can be obtained from Planck's formula;

$$\frac{hc}{k\lambda T} << 1$$

$$e^{\frac{hc}{k\lambda T}} = 1 + \frac{hc}{k\lambda T} + \frac{1}{2}\left(\frac{hc}{k\lambda T}\right)^2 + \cdots$$

Drop the second order and higher order terms, and RJ formula could be obtained.

(2) For smaller wavelength of blackbody radiation, the Wein's formula can be achieved also from Planck's experience formula;

$$\frac{1}{e^{\frac{hc}{\lambda kT}} - 1} \approx e^{-\frac{hc}{\lambda kT}}$$

Then Wein's formula could be obtained

(3) Integrating Planck's formula with respect to wavelength, the Stefan and Boltzmann's law can be obtained as well.

$$M(T) = \int_0^\infty M_\lambda(T)d\lambda = \sigma T^4$$ **, where σ is called Stefen constant.**

(4) Finally, according to the basic mathematical theory and differentiating the Planck's formula with respect to wavelength, Wien's displacement law can also be derived!

$$\frac{dM_\lambda(T)}{d\lambda} = 0$$

$$\Rightarrow \lambda_{\max} T = b$$

Planck's empirical formula matched all the different classical physics results obtained by the Maxwell electromagnetic theory, thermodynamics and statistics! However, no one knew why at that time. This phenomenon seemed unbelievable, incredible and even impossible, but is true!

In order to derive this formula theoretically, Planck proposed a brave hypothesis which is also incredible.

<u>Planck's Hypotheses:</u>

- **The molecules and atoms composing the blackbody concave can be regarded as the linear harmonic oscillator with electrical charge;**
- **The oscillators can only be in a special energy state. All these energies must be the integer multiples of a smallest energy ($\varepsilon_0 = h\nu$). Therefore the energies of the oscillators are $E = n\, h\nu$ with n = 1, 2, 3, …**

Using the hypothesis and classical physics, Planck arrived at his experience formula in two months later. The correct result shows that Planck's hypothesis is correct!

Quantum theory and modern physics was founded by these hypotheses!

2.6 Planck-Einstein Energy Quantization Law:

$$E = h\nu = \frac{hc}{\lambda}$$

Quantum energy (E), Planck constant (h), Frequency (ν)

$$h = 6.626 \times 10^{-34} J \cdot s$$
$$= 4.136 \times 10^{-15} eV \cdot s$$

$$1eV = 1.602 \times 10^{-19} J$$
$$1J = 6.242 \times 10^{18} eV$$

Example: **Calculate the photon energies for the following types of electromagnetic radiation: (a) a 600kHz radio wave; (b) the 500nm (wavelength of) green light; (c) a 0.1 nm (wavelength of) X-rays.**

Solution: (a) for the radio wave, we can use the Planck-Einstein law directly

$$E = h\nu = 4.136 \times 10^{-15} eV \cdot s \times 600 \times 10^{3} Hz$$
$$= 2.48 \times 10^{-9} eV$$

(b) The light wave is specified by wavelength, we can use the law explained in wavelength:

$$E = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6} eV \cdot m}{550 \times 10^{-9} m} = 2.26eV$$

(c) For X-rays, we have

$$E = \frac{hc}{\lambda} = \frac{1.241 \times 10^{-6} eV \cdot m}{0.1 \times 10^{-9} m} = 1.24 \times 10^{4} eV = 12.4keV$$

Therefore you can see that the higher frequency corresponds to the higher energy. The X-rays have quite high energy, so they have high power of penetration.

Here we emphasize that the particle properties of light and the photon will be defined. As we know the light is electromagnetic waves and it has the properties of waves.

PlPanck associated the energy quanta only with the light emission in the cavity walls and Einstein extended them to the absorption of radiation in his explanation of the *photoelectric effect*.

2.7 Photoelectric effect

The quantum nature of light had its origin in the theory of thermal radiation and was strongly reinforced by the discovery of the photoelectric effect.

In figure 2.1, a glass tube contains two electrodes of the same material, one of which is irradiated by light. The electrodes are connected to a battery and a sensitive current detector measures the current flow between them.

The current flow is a direct measure of the rate of emission of electrons from the irradiated electrode.

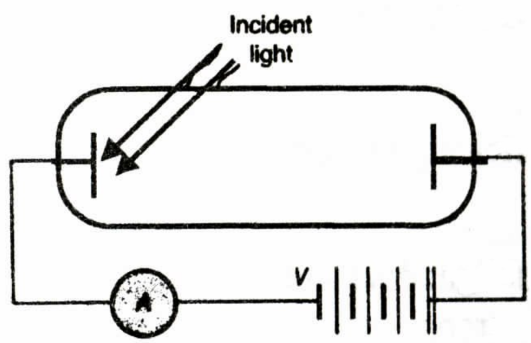


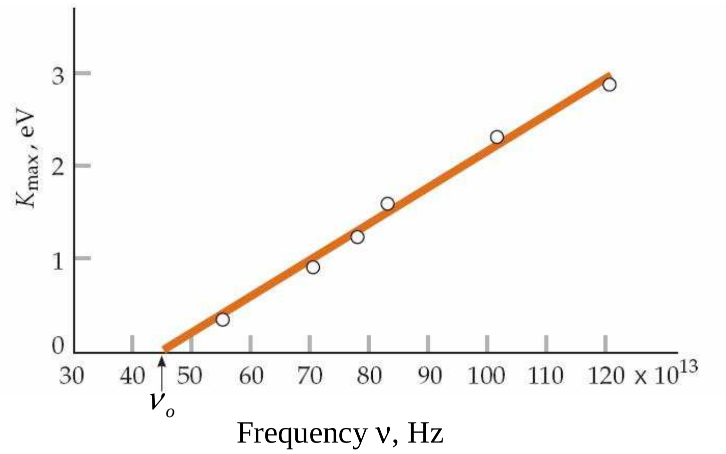
Fig. 2.1 Apparatus to investigate the photoelectric effect that was first found in 1887 by Hertz.

The electrons in the electrodes can be ejected by light and have a certain amount of kinetic energy.Now we change:

1. **the frequency and intensity of light,**
2. **the electromotive force (e.m.f. or voltage),**
3. **the nature of electrode surface.**

It is found that:

1. **For a given electrode material, no photoemission exists at all below a certain frequency of the incident light. When the frequency increases, the emission begins at a certain frequency. The frequency is called threshold frequency (ν_o) of the material. The threshold frequency has to be measured in the existence of e.m.f. (electromotive force) as at such a case the photoelectrons have no kinetic energy to move from the cathode to anode. Different electrode material has different threshold frequency.**



2. **The rate of electron emission is directly proportional to the intensity of the incident light.**
3. **Increasing the intensity of the incident light does not increase the kinetic energy of the photoelectrons.**
4. **There is no measurable time delay between irradiating the electrode and the emission of photoelectrons, even when the light is of very low intensity. As soon as the electrode is irradiated, photoelectrons are ejected.**
5. **The photoelectric current is deeply affected by the nature of the electrodes and chemical contamination of their surface.**

$$K_{\max} = eV$$

(1) In 1905, Einstein solved the photoelectric effect problem by applying the Planck's hypothesis. He pointed out that Planck's quantization hypothesis applied not only to the emission of radiation by a material object but also to its transmission and its absorption by another material object. The light is not only electromagnetic waves but also a quantum. All the effects of photoelectric emission can be readily explained from the following assumptions: The photoemission of an electron from a cathode occurs when an electron absorbs a photon of the incident light;

(2) The photon energy is calculated by the Planck's quantum relationship: E = hν.

(3) The minimum energy is required to release an electron from the surface of the cathode. The minimum energy is the characteristic of the cathode material and the nature of its surface. It is called work function (Φ).

The equation for the photoelectric emission can be written out by supposing the photon energy is completely absorbed by the electron. After this absorption, the kinetic energy of the electron should have the energy of the photon. If this energy is greater than the work function of the material, the electron should become a photoelectron and jumps out of the material and probably have some kinetic energy.

Therefore we have the equation of photoelectric effect:

$$K_{\max} = h\nu - \Phi$$

Where ($K_{\max}$) is the photoelectron kinetic energy , (*h*) is Planck's constant, (ν) is the frequency of the incident light and Φ is the work function.

$$K_{\max} = \frac{1}{2}\mathrm{mv}^2 \text{ and } \Phi = h\nu_o$$

$$hc = (4.136\times10^{-15}\ \mathrm{eV.s})(3\times10^{8}\,\mathrm{m/s}) = 1.240\times10^{-6}\ \mathrm{eV.m}$$

Or

$$hc = 1240 \quad \mathrm{eV.nm}$$

Example: **Ultraviolet light of wavelength 150nm falls on a chromium electrode. Calculate the maximum kinetic energy and the corresponding velocity of the photoelectrons (the work function of chromium is 4.37eV).**

Solution: **using the equation of the photoelectric effect, it is convenient to express the energy in electron volts. The photon energy is**

$$E = h\nu = \frac{hc}{\lambda} = \frac{1.241\times10^{-6}\,eV\cdot m}{150\times10^{-9}\,m} = 8.27eV$$

$$K_{\max} = h\nu - \Phi$$

$$\Rightarrow \frac{1}{2}\mathrm{mv}^2 = (8.27-4.37)eV = 3.90eV$$

$$1eV = 1.602\times10^{-19}\,J = 1.602\times10^{-19}\,N\cdot m = 1.602\times10^{-19}\,kg\cdot m^2\cdot s^{-2}$$

$$\frac{1}{2}\mathrm{mv}^2 = 3.90eV = 3.90\times1.602\times10^{-19}\,kg\cdot m^2\cdot s^{-2}$$

$$\mathrm{v} = \sqrt{\frac{2\times3.90eV}{m}} = \sqrt{\frac{12.496\times10^{-19}}{9.11\times10^{-31}}} = 1.17\times10^{6}\,m/s$$

2.8 Compton effect

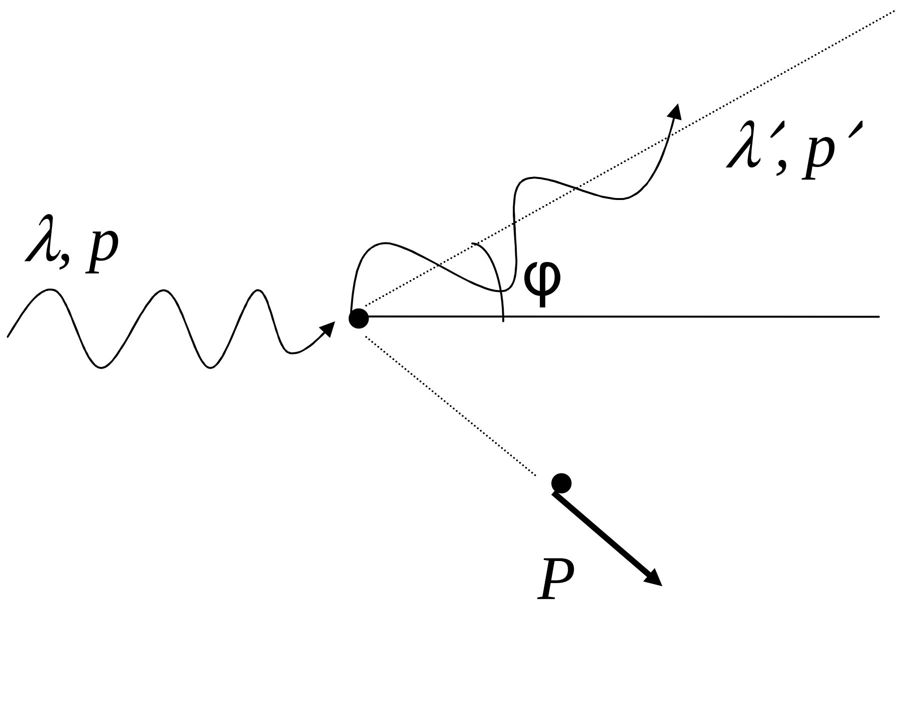
A phenomenon called Compton scattering, first observed in 1924 by Compton, provides additional direct confirmation of the quantum nature of electromagnetic radiation. When X-rays impinges on matter, some of the radiation is scattered, just as the visible light falling on a rough surface undergoes diffuse reflection. Observation shows that some of the scattered radiation has smaller frequency and longer wavelength than the incident radiation, and that the change in wavelength depends on the angle through which the radiation is scattered. Specifically, if the scattered radiation emerges at an angle φ with the respect to the incident direction, and if λ and λ' are the wavelength of the incident and scattered radiation, respectively, it is found that

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\varphi) = 2\frac{h}{mc}\sin^2\left(\frac{\varphi}{2}\right)$$

where *m is the electron mass.* $\Delta\lambda = \lambda' - \lambda$

$\lambda_c = \frac{h}{mc} = 0.00243 \ \ nm$ **called Compton wavelength**

In figure 2.1, the electron is initially at rest with incident photon of wavelength λ and momentum *p*; scattered photon with longer wavelength λ' and momentum *p′* and recoiling electron with momentum P. The direction of the scattered photon makes an angle φ with that of the incident photon, and the angle between p and p′ is also φ.



Home work of Ch.2:

Q1. Calculate the photon energies in (eV) for light of wavelengths 400 nm (violet) and 700 nm (red).

Q2. An X-ray photon of wavelength 6 pm makes a head-on collision with electron so that it is scattered by an angle of 180 ^{o}. (a) What is the change in wavelength of the photon? (b) What is the kinetic energy of the recoiling electron?