* Trigonometric integrals:-

* products of sines and cosines:-

$\int \sin^m x \cos^n x \, dx$

Case 1: m and n are both odd.

Case 2: m is odd and n is even.

Case 3: m is even and n is odd.

Case 4: m and n are both even.

Case 1: find $\int \cos^3 x \sin^5 x \, dx$

we write m or n as $n = 2K+1 \Rightarrow \cos^{2K+1} x = (\cos^2 x)^K \cos x$

and we use the identity $\cos^2 x = 1 - \sin^2 x$

sol/

$= \int \cos^2 x \sin^5 x \cos x \, dx \qquad (\cos^2 x = 1 - \sin^2 x)$

$= \int (1 - \sin^2 x) \sin^5 x \cos x \, dx = \int (\sin^5 x - \sin^7 x) \cos x \, dx$

let $u = \sin x$, $du = \cos x \, dx$

$= \int (u^5 - u^7) \, du = \frac{u^6}{6} - \frac{u^8}{8} + c = \frac{\sin^6 x}{6} + \frac{\sin^8 x}{8} + c$

Case 2: Find $\int \sin^3 x \cos^2 x \, dx$

we write m as $m = 2K+1 \Rightarrow \sin^m x = \sin^{2K+1} x = (\sin^2 x)^K \sin x$

and we use the identity $\sin^2 x = 1 - \cos^2 x$

Sol/ 19A

$\int \sin^3 x \cos^2 x \, dx = \int \sin^2 x \cos^2 x \sin x \, dx \quad (\sin^2 x = 1-\cos^2 x)$

$= \int (1-\cos^2 x)\cos^2 x \sin x \, dx = \int (\cos^2 x - \cos^4 x)\sin x \, dx$

let $u = \cos x$, $du = -\sin x \, dx$

$\ominus\int(\cos^2 x - \cos^4 x)\ominus\sin x \, dx = -\int (u^2 - u^4)\, du = -\left(\frac{u^3}{3} - \frac{u^5}{5}\right)+c$

$= \left(\frac{u^5}{5} - \frac{u^3}{3}\right)+c = \frac{\cos^5 x}{5} - \frac{\cos^3 x}{3} + c$

Case 3: find $\int \sin^2 x \cos^3 x \, dx$

we write $n = 2k+1 \Rightarrow \cos^{2k+1} x = (\cos^2 x)^k \cos x$ and we use the identity $(\cos^2 x = 1-\sin^2 x)$

$= \int \sin^2 x \cos^2 x \cos x \, dx \qquad (\cos^2 x = 1-\sin^2 x)$

$= \int \sin^2 x (1-\sin^2 x)\cos x \, dx = \int (\sin^2 x - \sin^4 x)\cos x \, dx$

let $u = \sin x$, $du = \cos x \, dx$

$= \int (u^2 - u^4)\, du = \frac{u^3}{3} - \frac{u^5}{5} + c = \frac{\sin^3 x}{3} - \frac{\sin^5 x}{5} + c$

Case 4: find $\int \sin^2 x \cos^4 x$

we substitute $\sin^2 x = \frac{1-\cos 2x}{2}$, $\cos^2 x = \frac{1+\cos 2x}{2}$

Sol/

$= \int \frac{1-\cos 2x}{2} * \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{8}\int (1-\cos 2x)(1+\cos 2x)^2 dx$

$= \frac{1}{8}\int (1-\cos 2x)(1+2\cos 2x + \cos^2 2x)\, dx = \int (1+$

$= \frac{1}{8}\int (1+2\cos 2x + \cos^2 2x - \cos 2x - 2\cos^2 2x - \cos^3 2x)\, dx$

$$= \frac{1}{8}\int (1 + \cos 2x - \cos^2 2x - \cos^3 2x)\, dx$$

$$\int \cos^2 2x\, dx = \int \frac{1+\cos 4x}{2}\, dx = \frac{1}{2}\int (1+\cos 4x)\, dx$$

$$= \frac{1}{2}\int dx + \frac{1}{2}\int \cos 4x\, dx = \frac{1}{2}x + \frac{1}{2} * \frac{1}{4}\int \cos 4x * 4\, dx$$

$$= \frac{x}{2} + \frac{1}{8}\sin 4x = \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right)$$

$$\int \cos^3 2x\, dx = \int \cos^2 2x \cos 2x\, dx = \frac{1}{2}\int (1-\sin^2 2x)\cos 2x * 2\, dx$$

let $u = \sin 2x$, $du = \cos 2x * 2\, dx$

$$= \frac{1}{2}\int (1-u^2)\, du = \frac{1}{2}\left(u - \frac{u^3}{3}\right) = \frac{1}{2}\left(\sin 2x - \frac{\sin^3 2x}{3}\right)$$

$$\therefore \int \sin^2 x \cos^4 x\, dx = \frac{1}{8}\left[x + \frac{1}{2}\sin 2x - \frac{1}{2}\left(x + \frac{1}{4}\sin 4x\right) - \frac{1}{2}\left(\sin 2x - \frac{\sin^3 2x}{3}\right)\right] + c$$

$$= \frac{1}{8}\left[x + \frac{1}{2}\sin 2x - \frac{1}{2}x - \frac{1}{8}\sin 4x - \frac{1}{2}\sin 2x + \frac{1}{6}\sin^3 2x\right] + c$$

$$= \frac{1}{8}\left[\frac{x}{2} - \frac{1}{8}\sin 4x + \frac{1}{6}\sin^3 2x\right] + c$$

$$= \frac{1}{16}\left[x - \frac{1}{4}\sin 4x + \frac{1}{3}\sin^3 2x\right] + c$$

* Eliminating square roots :-
we use the identity $\cos^2 x = \frac{1+\cos 2x}{2}$

ex/ find $\int \sqrt{1+\cos 4x}\, dx$

Sol/

$\cos^2\theta = \frac{1+\cos 2\theta}{2} \Rightarrow 1+\cos 2\theta = 2\cos^2\theta \;, \; 2\theta = 4x$

$\int \sqrt{1+\cos 4x}\, dx = \int \sqrt{2\cos^2 2x}\, dx = \sqrt{2}\int \sqrt{\cos^2 2x}\, dx$

$= \sqrt{2} * \frac{1}{2}\int \cos 2x \, 2dx = \sqrt{2} * \frac{\sin 2x}{2} + c = \frac{\sin 2x}{\sqrt{2}} + c$

* Integrals of powers of $(\tan x)$ and $(\sec x)$
we use the identity $\sec^2 x - \tan^2 x = 1 \Rightarrow \tan^2 x = \sec^2 x - 1$

ex/ find $\int \tan^4 x\, dx$

Sol/

$= \int \tan^2 x \cdot \tan^2 x\, dx = \int \tan^2 x\,(\sec^2 x - 1)\, dx = \int (\tan^2 x \sec^2 x - \tan^2 x) dx$

$= \int \tan^2 x \sec^2 x\, dx - \int \tan^2 x\, dx = \int \tan^2 x \sec^2 x\, dx - \int (\sec^2 x - 1)\, dx$

$= \frac{\tan^3 x}{3} - \tan x + x + c$

ex/ Evaluate $\int \sec^3 x\, dx$ (we integrate by parts)

Sol/

$= \int \sec x \cdot \sec^2 x\, dx$ (let $u = \sec x$, $dv = \sec^2 x\, dx$, $du = \sec x \tan x\, dx$
$v = \tan x$)

$= \sec x \tan x - \int \tan x\,(\sec x \tan x)\, dx$

$= \sec x \tan x - \int \tan^2 x \sec x \, dx$

$= \sec x \tan x - \int (\sec^2 x - 1) \sec x \, dx$

$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$2\int \sec^3 x \, dx = \sec x \tan x + \int \sec x * \frac{\sec x + \tan x}{\sec x + \tan x} \, dx$

$2\int \sec^3 x \, dx = \sec x \tan x + \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} \, dx$

$2\int \sec^2 x \, dx = \sec x \tan x + \ln|\sec x + \tan x|$

$\therefore \int \sec^3 x \, dx = \frac{\sec x \tan x + \ln|\sec x + \tan x|}{2} + c$

$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln|\sec x + \tan x| + c$

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* products of Sines and cosines:-

The integrals:

$$\int \sin mx \sin nx \, dx, \int \sin mx \cos nx \, dx, \int \cos mx \cos nx \, dx$$

we use the identities:

$\sin mx \sin nx = \frac{1}{2} (\cos(m-n)x - \cos(m+n)x)$

$\sin mx \cos nx = \frac{1}{2} (\sin(m-n)x + \sin(m+n)x)$

$\cos mx \cos nx = \frac{1}{2} (\cos(m-n)x + \cos(m+n)x)$

$\sin mx \sin nx = \frac{1}{2} (\cos(m-n)x - \cos(m+n)x)$

$\sin mx \cos nx = \frac{1}{2} (\sin(m-n)x + \sin(m+n)x)$

$\cos mx \cos nx = \frac{1}{2} (\cos(m-n)x + \cos(m+n)x)$

ex/ Evaluate $\int \sin 3x \cos 5x \, dx$ 4A

Sol/

$$\sin 3x \cos 5x = \frac{1}{2}(\sin(3-5)x + \sin(3+5)x)$$

$$= \frac{1}{2}(\sin(-2x) + \sin 8x)$$

$$\int \sin 3x \cos 5x \, dx = \frac{1}{2}\int (\sin(-2x) + \sin 8x) \, dx$$

$$= \frac{1}{2} * \frac{-1}{2} \int \sin(-2x) * -2 \, dx + \frac{1}{2} * \frac{1}{8} \int \sin 8x * 8 \, dx$$

$$= \frac{-1}{4}(-\cos(-2x)) + \frac{1}{16}(-\cos 8x) + c$$

$$= \frac{1}{4}\cos(-2x) - \frac{1}{16}\cos 8x + c$$