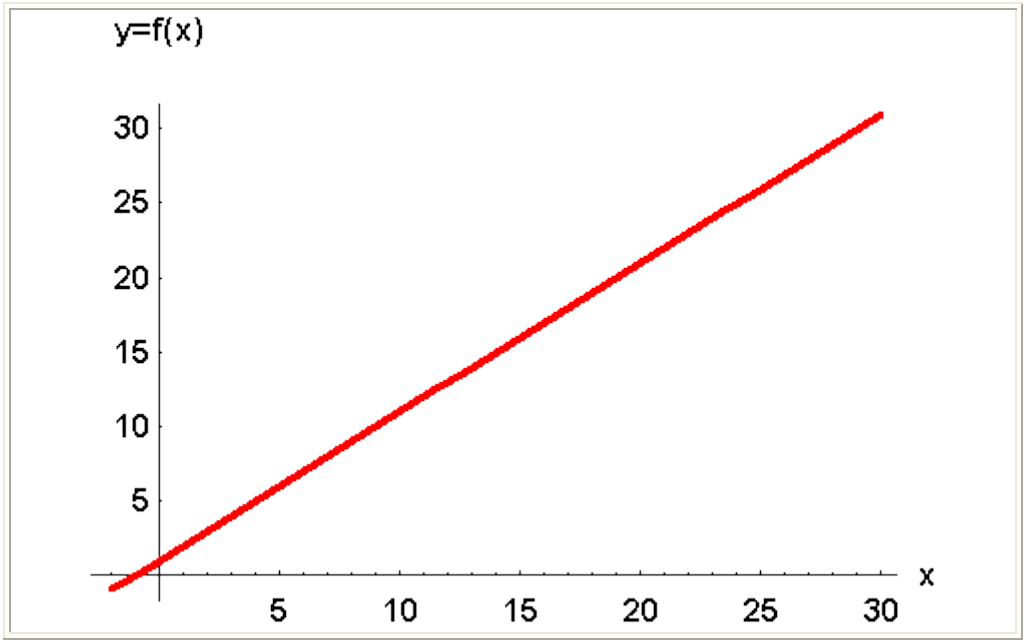
Function

Before we review exponential and logarithmic functions, let's review the definition of a function and the graph of a function. A function is just a rule. The rule links one number to a second number in an orderly and specific manner. All the points on the graph of a function are made up of two parts: (a number, and the function value at that number). For example, the number of hours worked in a week could be the first number, and the salary for the week could be the function value. If an hourly salary is $7.00, then the rule would be 7 times the number of hours worked.

You could identify a point on the graph of a function as (x,y) or (x, f(x)). You may have only one function value for each x number.

If the points (2, 3), (4, 5), (10, 11), and (25, 26) are located on the graph of a function, you could easily figure out a corresponding rule. To get the function value, you just add 1 to the first number. The rule is $f(x) = x + 1$.

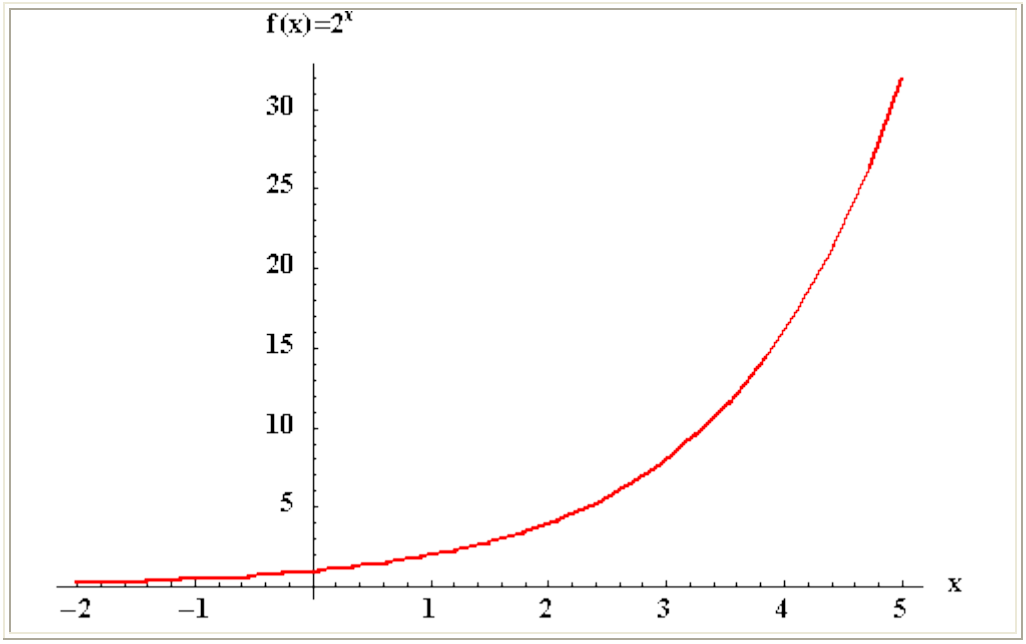


The points (3, 8) and (3, 18) could not be points on the graph of a function because there are two different function values for the same x value

Definition of Exponential Function

The exponential function f with base a is denoted by $f(x) = a^x$, where $a \neq 1$, and x is any real number. The function value will be positive because a positive base raised to any power is positive. This means that the graph of the exponential function $f(x) = a^x$ will be located in quadrants I and II.

For example, if the base is 2 and x = 4, the function value f(4) will equal 16. A corresponding point on the graph of $f(x) = 2^x$ would be (4, 16).

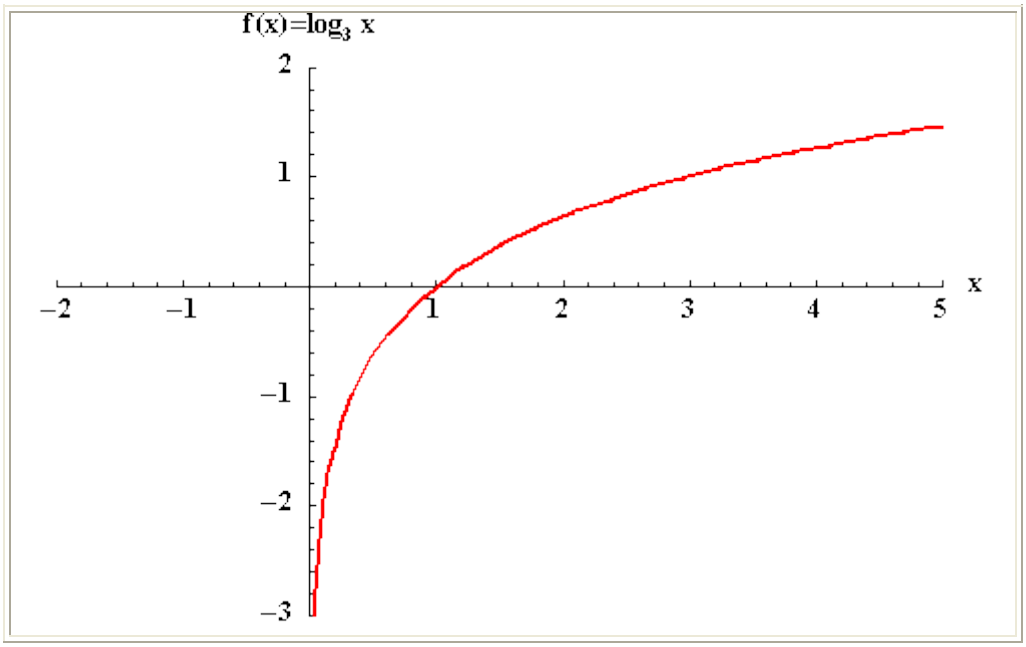


Definition of Logarithmic Function

For $x > 0$, $a > 0$, and $a \neq 1$, we have

$$f(x) = \log_a(x) \text{ if and only if } a^{f(x)} = x .$$

Since $x > 0$, the graph of the above function will be in quadrants I and IV.



- **PROPERTIES OF LOGARITHMS**

- **Property 1:** $Log_a 1 = 0$ because $a^0 = 1$.

- **Example 1:** In the equation $14^0 = 1$, the base is 14 and the exponent is 0. Remember that a logarithm is an exponent, and the corresponding logarithmic equation is $\log_{14} 1 = 0$ where the 0 is the exponent.
- **Example 2:** In the equation $\left(\frac{1}{2}\right)^0 = 1$, the base is $\frac{1}{2}$ and the exponent is 0. Remember that a logarithm is an exponent, and the corresponding logarithmic equation is $\log_{\frac{1}{2}} 1 = 0$.
- **Example 3:** Use the exponential equation $x^0 = 1$ to write a logarithmic equation. The base x is greater than 0 and the exponent is 0. The corresponding logarithmic equation is $\log_x 1 = 0$.
- **Property 2:** $\log_a a = 1$ because $a^1 = a$.
- **Example 4:** In the equation $3^1 = 3$, the base is 3, the exponent is 1, and the answer is 3. Remember that a logarithm is an exponent, and the corresponding logarithmic equation is $\log_3 3 = 1$.
- **Example 5:** In the equation $87^1 = 87$, the base is 87, the exponent is 1, and the answer is 87. Remember that a logarithm is an exponent, and the corresponding logarithmic equation is $\log_{87} 87 = 1$.
- **Example 6:** Use the exponential equation $p^1 = p$ to write a logarithmic equation. If the base p is greater than 0, then $\log_p p = 1$.
- **Property 3:** $\log_a a^x = x$ because $a^x = a^x$.
- **Example 7:** Since you know that $3^4 = 3^4$, you can write the logarithmic equation with base 3 as $\log_3 3^4 = 4$.
- **Example 8:** Since you know that $13^4 = 13^4$, you can write the logarithmic equation with base 13 as $\log_{13} 13^4 = 4$.
- **Example 9:** Use the exponential equation $4^2 = 16$ to write a logarithmic equation with base 4. You can convert the exponential equation
- $4^2 = 16$to the logarithmic equation $\log_4 16 = 2$. Since the 16 can be written as

- 4^2, the equation $\log_4 16 = 2$ can be written $\log_4 4^2 = 2$.
- The above rules are the same for all positive bases. The most common bases are the base 10 and the base *e*. Logarithms with a base 10 are called **common logarithms**, and logarithms with a base *e* are **natural logarithms**. On your calculator, the base 10 logarithm is noted by **log**, and the base *e* logarithm is noted by **ln**.
- There are an infinite number of bases and only a few buttons on your calculator. You can convert a logarithm with a base that is not 10 or *e* to an equivalent logarithm with base 10 or e. If you are interested in a discussion on how to change the bases of a logarithm.