Failure of Brittle Materials under static Loading

Brittle materials fracture rather than yield. Brittle fracture in tension is considered to be due to the normal tensile stress alone and thus the max. normal-stress theory is applicable in this case. Brittle fracture in compression is due to some combination of normal compressive stress and shear stress and required different theory of failure.

Even and Uneven Brittle Materials

If the material compressive strength equal to their tensile strength, then material is called even material such as fully hardened tool steel (brittle). Many cast materials, such as gray cast iron, are brittle but have compressive strengths much greater than their tensile strengths. These are called uneven materials

Fig. ③ shows Mohr's circles for both compression and tensile tests of even material and an uneven material. The lines tangent to these circles constitute failure lines for all combinations of applied stresses between the circles.

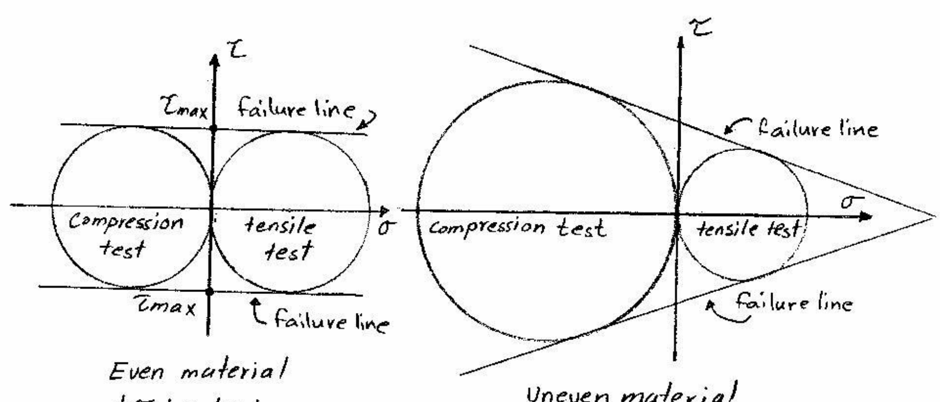
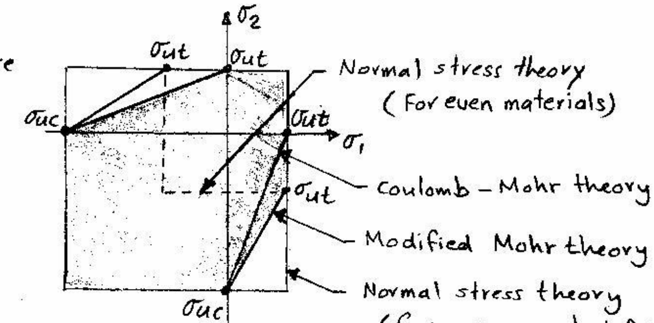


Fig. ③

Theories of Brittle Failure

Note: The modified-Mohr theory is the preferred failure theory for uneven brittle materials in static loading



Modified-Mohr theory

$$N = \frac{\sigma_{ut}}{\sigma_1} \quad \text{if } \sigma_1 \geqslant \sigma_2 \geqslant 0$$
$$\text{or } \sigma_1 \geqslant 0 \geqslant \sigma_2 \text{ and } \left|\frac{\sigma_2}{\sigma_1}\right| \leqslant 1$$

$$N = \frac{\sigma_{ut}\sigma_{uc}}{\sigma_{uc}\sigma_1 - \sigma_{ut}(\sigma_1 + \sigma_2)} \quad \text{if } \sigma_1 \geqslant 0 \geqslant \sigma_2 \text{ and } \left|\frac{\sigma_2}{\sigma_1}\right| > 1$$

$$N = \frac{|\sigma_{uc}|}{\sigma_2} \quad \text{if } 0 \geqslant \sigma_1 \geqslant \sigma_2$$

Ex. Repeat the example in page ⑧, using brittle material with $\sigma_{ut} = 52500$ psi, $\sigma_{uc} = -164000$ psi instead of ductile material. Find the safety factor using Modified-Mohr theory.

► point A (bending + torsion)

$\sigma_x = 18108$ psi, $\tau_{xy} = 12072$ psi
$\sigma_1 = 24144$ psi, $\sigma_2 = -6036$ psi
} as found before

since $\sigma_1 \geqslant 0 \geqslant \sigma_2$, and $\left|\frac{\sigma_2}{\sigma_1}\right| \leqslant 1 \Rightarrow N = \frac{\sigma_{ut}}{\sigma_1} = \frac{52500}{24144} = 2.2$

► point B (transverse shear + torsion)

$\tau_{max} = 12827$ psi

$\sigma_1 = 12827$ psi, $\sigma_2 = -12827$ psi

since $\sigma_1 > 0 > \sigma_2$, and $\left|\frac{\sigma_2}{\sigma_1}\right| = 1 \Rightarrow N = \frac{\sigma_{ut}}{\sigma_1} = \frac{52500}{12827} = 4.1$

stress concentration factors

The condition where high localized stresses are produced as a result of an abrupt change in geometry is called the stress concentration. The abrupt change in form or discontinuity occur is such frequently encountered raises as holes, notches, Keyways, thread, grooves, and fillets.

A geometric or theoretical stress concentration factor K_t is used to relate the maximum stress at the discontinuity to the nominal stress. The factor is defined by

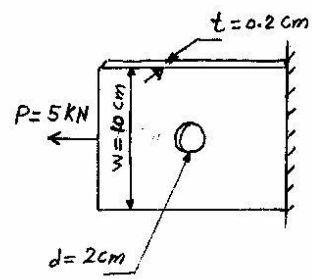
$$K_t = \frac{\sigma_{max}}{\sigma_{nom}} \quad \text{or} \quad K_{ts} = \frac{\tau_{max}}{\tau_{nom}}$$

Here the nominal stresses are the stresses that would occur if the abrupt change in the cross section didnot exist or had no infuence on the stress distribution

In static loading, the stress concentration factor is ignored for all ductile materials (why) but its very important factor for brittle materials (except the cray cast iron, why).

In dynamic loading, the stress concentration faction must be considered regardless of whether the material response is brittle or ductile.

Ex. For a plate that made from a brittle material alloy shown below, Find the max. stress that occur.



$Area = (10-2)\times 0.2 = 1.6\ cm^2 = 1.6\times10^{-4}\ m^2$

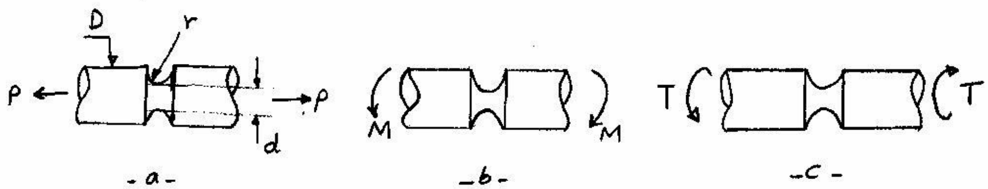
From Appendix (E), Fig. E-13

$\frac{d}{W} = \frac{2}{10} = 0.2 \Rightarrow K_t = 2.5$

$\sigma_{nom} = \frac{P}{A} = \frac{5\times10^3}{1.6\times10^{-4}} = 31.25\ MPa$

$\sigma_{max} = K_t\,\sigma_{nom} = 2.5\times31.25 = 78.125\ MPa$

Ex. Find the stress concentration factor of grooved shafts that loaded as shown , take $\frac{D}{d} = 2$, $\frac{r}{d} = 0.15$



a) From Appendix E , Fig. E-4

$K_t \cong A \left(\frac{r}{d}\right)^b$

$A = 0.99383$, $b = -0.38231$ at $\frac{D}{d} = 2$

$K_t = 2.052$

b) From Appendix E , Fig. E-5

$A = 0.93619$, $b = -0.33066$ at $\frac{D}{d} = 2$

$K_t = 1.753$

c) From Appendix E , Fig. E-6

$A = 0.89035$, $b = -0.24075$ at $\frac{D}{d} = 2$

$K_t = 1.405$