

$$3 - \frac{1}{2} = \frac{6-1}{2} = \frac{5}{2}$$

The discharge ratio

$$\frac{Q_p}{Q_m} = \frac{l_p^3/t_p}{l_m^3/t_m} = \frac{l_p^3}{l_m^3} \cdot \frac{t_m}{t_p} = \frac{l_p^3}{l_m^3} \cdot \frac{t_m}{t_m \sqrt{\lambda_L}}$$

$$= \lambda_L^3 \cdot \frac{1}{\sqrt{\lambda_L}} = \lambda_L^{5/2}$$

$$\therefore \frac{Q_p}{Q_m} = \lambda_L^{5/2} \Rightarrow \boxed{Q_p = Q_m \lambda_L^{5/2}}$$

The Force ratio

$$\frac{F_p}{F_m} = \frac{\gamma_p l_p l_p^2}{\gamma_m l_m l_m^2} = \frac{\gamma_p l_p l_p^2}{\gamma_m l_m l_m^2} \Rightarrow \frac{F_p}{F_m} = \lambda_L^3$$

$$\frac{F_p}{F_m} = \lambda_L^3 \Rightarrow \boxed{F_p = F_m \lambda_L^3}$$

Ship Model Tests 80

The resistance is the sum of the wave resistance (Free-surface or Fr-phenomenon) and the viscous resistance (Re-Phenomenon).

If $Re_m = Re_p$ and $\rho_p = \rho_m, \mu_p = \mu_m$

$$\frac{V_m l_m \rho_m}{\mu_m} = \frac{V_p l_p \rho_p}{\mu_p} \Rightarrow V_m = V_p \frac{l_p}{l_m} > V_p$$

If $Fr_m = Fr_p$ and $g_m = g_p$

$$\frac{V_m^2}{g_m l_m} = \frac{V_p^2}{g_p l_p} \Rightarrow V_m = V_p \sqrt{\frac{l_m}{l_p}} < V_p$$

Hence $Re =$ criterion give higher V_m than $Fr =$ criterion, whereas $Fr =$ criterion give lower V_m than $Re =$ criterion.

The procedure is to model for the phenomenon that is most difficult to predict analytically and to account for the other resistance by analytical means. Since the wave resistance is the most difficult problem, the model is tested according

to the Fr-criterion and viscous resistance, is accounted for analytically.

Procedure :-

$$F_{DT} = F_{DV} + F_{DW}$$

total drag viscous drag wave drag

hence

$$F_{DTm} = F_{DVm} + F_{Dwm}$$

$$F_{DTP} = F_{DVP} + F_{DWP}$$

- ① First make model tests according to the Fr-criterion and the total model drag (F_{DTm}) is measured.
- ② Estimate the viscous drag (F_{DVm}) by analytical calculations
- ③ Calculate $F_{Dwm} = F_{DTm} - F_{DVm}$
- ④ Using Fr-criterion calculate F_{DWP} from F_{Dwm}
($F_{DWP} = F_{Dwm} \lambda_e^3$)
- ⑤ Calculate F_{DVP} analytically
- ⑥ Calculate $F_{DTP} = F_{DVP} + F_{DWP}$

EX A (10 m) model of an ocean tanker (500 m) long is dragged through fresh water at (3 m/s) with a total measured resistance of (103 N). The surface drag coefficient (C_f) for model and prototype are (0.245) and (0.0147) respectively, the equation ($F = C_f A V^2$). The wetted surface area of the model is (20 m²). Find the total drag on the tanker and the power required at the prop shaft assuming an efficiency of (90%) for the prop.

Sol

step ① $F_{Dm} = C_{f_m} A_m V_m^2 = 0.245 \times 20 \times 3^2 = \underline{44.1 \text{ N}}$

step ② $F_{DTP} = F_{DTM} - F_{Dm} = 103 - 44.1 = \underline{58.9 \text{ N}}$

using Fr. criterion

step ③ $F_{DTP} = F_{Dm} \lambda_L^3 = 58.9 \times \left(\frac{500}{10}\right)^3 = \underline{7362500 \text{ N}}$

or

$$V_p = V_m \sqrt{\lambda_L} = 3 \sqrt{\frac{500}{10}} \Rightarrow V_p = 21.2 \text{ m/s}$$

$$\frac{F_{DTP}}{F_{Dm}} = \frac{\rho_p l_p^2 V_p^2}{\rho_m l_m^2 V_m^2} = \frac{(500)^2 \times (21.2)^2}{(10)^2 \times (3)^2} \Rightarrow \underline{F_{DTP} = 7362500 \text{ N}}$$

$$\frac{A_p}{A_m} = \frac{l_p^2}{l_m^2} = \frac{(500)^2}{(10)^2} \Rightarrow A_p = 20 \times 2500 = \underline{50000 \text{ m}^2}$$

step ④ $F_{DTP} = C_{f_p} A_p V_p^2 = 0.0147 \times 50000 \times (21.2)^2 = \underline{330338.4 \text{ N}}$

step ⑤ $F_{DTP} = F_{DTP} + F_{DTP} \Rightarrow$

$$\therefore F_{DTP} = 7362500 + 330338.4 = \underline{7692838.4 \text{ N}}$$

$$\text{Shaft power} = \frac{\text{output power}}{\eta} = \frac{F_{DTP} \times V_p}{\eta} = \frac{7692838.4 \times 21.2}{0.9}$$

$$\therefore \text{shaft power} = 181,209,082.3 \text{ J} \\ = 181.209082 \text{ MJ} \quad \underline{\underline{\text{ans.}}}$$

EX A V-notch weir is vertical plate with a notch of angle ϕ cut into the top of it and placed across an open channel. The liquid in the channel is backed up and forced to flow through the notch. The discharge Q is some function of the elevation H of upstream liquid surface above the bottom of the notch. In addition, the discharge depends upon gravity and upon the velocity of approach V_0 to the weir. Determine the form of discharge equation.

Sol

$$F(Q, H, g, V_0, \phi) = 0$$

EX The losses $\Delta h/l$ per unit length of pipe in turbulent flow through a smooth pipe depend upon velocity V , diameter D , gravity g , dynamic viscosity μ and the density ρ . With dimensional analysis. Determine the general form of the equation.

Sol

$$F\left(\frac{\Delta h}{l}, V, D, \rho, \mu, g\right) = 0$$

EX A fluid flow situation depend upon the velocity V , the density ρ , several linear dimensions l_1, l_2 , pressure drop ΔP , gravity g , viscosity μ , surface tension σ and the bulk modulus of elasticity k . Apply dimensional analysis to these variable to find a set of π parameters

Sol

$$F(V, \rho, \mu, l_1, l_2, \Delta P, g, \sigma, k) = 0$$