Common Dimensionless parameters:

In a hypothetical Fluid Flow situations the pressure difference (ΔP) between two points in the Flow Field depends on ($V, \rho, l, l_1, l_2, \mu, k, \sigma, \Delta\gamma$) thus:

$$F(\Delta P, \rho, V, l, l_1, l_2, \mu, \Delta\gamma, \sigma, k) = 0.0$$

Dependent Variable $= \Delta P$

$n = 10$, $m = 3 \Rightarrow$ No of π's $= 7$

Chose (V, ρ, l) as the repeating variables, we obtain

$$f_1(\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7) = 0.0$$

$$f_1\left(\frac{\Delta P}{\rho V^2}, \frac{\rho l V}{\mu}, \frac{V}{\sqrt{K/\rho}}, \frac{\rho l V^2}{\sigma}, \sqrt{\frac{V^2}{l\frac{\Delta\gamma}{\rho}}}, \frac{l}{l_1}, \frac{l}{l_2}\right) = 0.0$$

or

$$\frac{\Delta P}{\frac{1}{2}\rho V^2} = F_2\left(\frac{\rho l V}{\mu}, \frac{V}{\sqrt{K/\rho}}, \frac{\rho l V^2}{\sigma}, \sqrt{\frac{V^2}{l\frac{\Delta\gamma}{\rho}}}, \frac{l}{l_1}, \frac{l}{l_2}\right) = 0$$

Inertia Force:

$$F_i = ma \Rightarrow F_i \propto \rho l^3 \left(\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial s}\right)$$

For steady flow $\frac{\partial V}{\partial t} = 0.0$

thus

$$F_i \propto \rho l^3 V \frac{\partial V}{\partial s} \Rightarrow \boxed{F_i \propto (\rho l^2 V^2)}$$

pressure Force:

$$F_P \propto \Delta P \times A \propto \Delta P l^2$$

$$C_P = \frac{F_P}{F_i} = \frac{\Delta P l^2}{\frac{1}{2}\rho l^2 V^2} \Rightarrow \boxed{C_P = \frac{\Delta P}{\frac{1}{2}\rho V^2}}$$ pressure cofficient

note:-

Euler number (E) is defined as $\boxed{E = \frac{V}{\sqrt{\Delta P/\rho}}}$

Viscous Forces:-

$$F_V \propto \tau A \propto \mu \frac{dV}{dy} A \propto \mu \frac{V}{l} l^2 \Rightarrow F_V \propto \mu l V$$

$$Re = \frac{F_i}{F_V} = \frac{\frac{1}{2}\rho l^2 V^2}{\mu l V} \Rightarrow \boxed{Re = \frac{\rho l V}{\mu}}$$ Reynold number

Note:-

① At high Re the effect of viscosity is small and the Re effect is negligible because the inertia effect is large. High Re means either (V) is large or μ is small or l is large.

Surface tension Force:

$$F_\sigma = \sigma l$$

$$W = \frac{F_i}{F_\sigma} = \frac{\rho l^2 V}{\sigma l} \Rightarrow \boxed{W = \frac{\rho l V^2}{\sigma}}$$ weber number

(W) is used when surface tension effects are predominant, such as capillary waves in channels.

Gravity Force:-

$$F_g \propto \Delta\gamma l^3$$

$$(Fr)^2 = \frac{F_i}{F_g} = \frac{\rho l^2 V^2}{\Delta\gamma l^3} = \frac{V^2}{l \frac{\Delta\gamma}{\rho}} = \frac{V^2}{g l} \Rightarrow \boxed{Fr = \frac{V}{\sqrt{g l}}}$$

Froud Number

(Fr) is used when there is density stratification such as between salt water and fresh water.

Elastic or Compressible Forces:-

$$F_c = \Delta\rho C l^2 = \rho V c l^2$$

$$c = \sqrt{\frac{E}{\rho}}$$ speed of sound

$$M = \frac{F_i}{F_c}$$

$$= \frac{\rho l^2 V^2}{\rho V c l^2} \Rightarrow \boxed{M = \frac{V}{c} = \frac{V}{\sqrt{K/\rho}}}$$ Mach Number

(M) is important when $M > 0.4$. (M) is a measure as the ratio of K.E of the flow to internal energy of the flow, since

$$M = \frac{V}{c} \propto \frac{V^2}{c^2} \propto \frac{V^2}{kRT} \propto \frac{K.E}{\text{Internal Energy}}$$

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thus:

$$C_p = f_2\left(Re, M, W, Fr, \frac{l}{l_1}, \frac{l}{l_2} \right)$$

Similitude:

It is the theory and art of predicting the prototype performance from model observation. It involves the application of the dimensionless numbers such as (Re, M, Fr, W) to predict prototype performance from model tests.

① Geometrical Similitude:

The shape of the model is similar to the prototype. The ratio between all corresponding lengths ~~and~~ including roughness between model and prototype are equal. The ratio (λ_l) is called the "scale ratio" or "Model ratio"

$$\lambda_l = \frac{l_p}{l_m} \quad \text{length ratio}$$

$$\lambda_A = \frac{A_p}{A_m} \quad \text{Area ratio}$$

$$\lambda_{\forall} = \frac{\forall_p}{\forall_m} \quad \text{Volume ratio}$$

② Kinematic similarity:

ⓐ The path of homologous moving particles are geometrically similar.

ⓑ The ratios of velocities and acceleration at homologous particles are equal.

Kinematic similarity implies geometric similarity not via versa.

$$\lambda_v = \frac{V_p}{V_m} = \frac{l_p/t_p}{l_m/t_m} = \frac{\lambda_l}{\lambda_t}$$

$$\lambda_a = \frac{a_p}{a_m} = \frac{l_p/t_p^2}{l_m/t_m^2} = \frac{\lambda_l}{\lambda_t^2}$$

$$\lambda_Q = \frac{Q_p}{Q_m} = \frac{l_p^3/t_p}{l_m^3/t_m} = \frac{\lambda_l^3}{\lambda_t}$$

③ Dynamic Similarity :-

This implies that in geometrically and kinematically similar systems at homologous points the ratio of corresponding forces between the systems are same.

The dimensional analysis produces dimensionless grouping which should be same in the prototype and model if dynamic similarity is to be obtained

$$\left(Re, Fr, W, M, \frac{l}{l_1}, \frac{l}{l_2}\right)_m = \left(Re, Fr, W, M, \frac{l}{l_1}, \frac{l}{l_2}\right)_p$$

Pipe Flow :-

In steady flow in pipe, viscous and inertia forces, only ones of consequence, hence

$$Re_p = Re_m$$

$$\boxed{\frac{V_p D_p \rho_p}{\mu_p} = \frac{V_m D_m \rho_m}{\mu_m}}$$

Open hydraulic Structure :-

Forces due to gravity (from changing on elevation of liquid surface) and inertia force are important, Thus

$$Fr_p = Fr_m$$

$$\frac{V_p^2}{g_p l_p} = \frac{V_m^2}{g_m l_m}$$

but $g_p = g_m$ & $\lambda_l = \frac{l_p}{l_m}$

then

$$\boxed{V_p = V_m \sqrt{\lambda_l}}$$

$$\lambda_t = \frac{t_p}{t_m} = \frac{l_p / V_p}{l_m / V_m} = \frac{l_p}{l_m} \frac{V_m}{V_p} = \frac{l_p}{l_m} \cdot \sqrt{\frac{l_m}{l_p}}$$

$$\therefore \lambda_t = \sqrt{\lambda_l} \Rightarrow \boxed{t_p = t_m \sqrt{\lambda_l}}$$