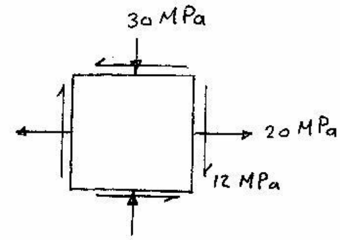
Ex. Estimate the factor of safety for a ductile materail having $\sigma_y = 100$ MPa, with the state of stress shown in the element.



$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}}$$

$\sigma_x = 20$ MPa , $\sigma_y = -30$ MPa , $\tau_{xy} = 12$ MPa

$\Rightarrow \sigma_1 = 22.73$ MPa

$\sigma_2 = -32.73$ MPa

① Using Normal stress theory

$$|\sigma_2| > |\sigma_1| \quad \Rightarrow N = \frac{\sigma_y}{|\sigma_2|} = \frac{100 * 10^6}{32.73 \times 10^6} = 3.055$$

② Using Tresca

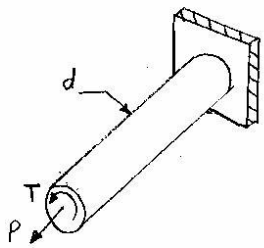
$$N = \frac{\sigma_y}{\sigma_1 - \sigma_2} = \frac{100 \times 10^6}{22.73 \times 10^6 - (-32.73 \times 10^6)} = 1.803$$

③ Von _ Mises

$$N = \frac{\sigma_y}{\left[\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2\right]^{\frac{1}{2}}} = \frac{100 \times 10^6}{(22.73 \times 10^6)^2 - (22.73 \times 10^6)(-32.73 \times 10^6) + (32.73 \times 10^6)^2}$$

$$= 2.07$$

Ex. A circular rod, constructed of a ductile material of tensile strength $\sigma_y = 300$ MPa, is subjected to a torque $T = 500\pi$ N.m and axial tensile load P. The diameter of the rod $d = 50$ mm. Using Tresca criterion with $N = 1.2$, Find allowable P.



$$\tau = \frac{16T}{\pi d^3} = \frac{16 \times 500\pi}{\pi(50\times10^{-3})^2}$$

$$= 64 \text{ MPa}$$

$$\sigma = \frac{P}{A} = \frac{4P}{\pi(50\times10^{-3})^2}$$

$$\sigma_1 = \frac{4P}{2\pi(50\times10^{-3})^2} + \left[\left(\frac{4P}{2\pi(50\times10^{-3})^2}\right)^2 + (64\times10^6)^2\right]^{\frac{1}{2}}$$

$$\sigma_2 = \frac{4P}{2\pi(50\times10^{-3})^2} - \left[\left(\frac{4P}{2\pi(50\times10^{-3})^2}\right)^2 + (64\times10^6)^2\right]^{\frac{1}{2}}$$

According to Tresca criterion

$$\sigma_1 - \sigma_2 = \frac{\sigma_y}{N}$$

$$2\left[\left(\frac{2P}{\pi(50\times10^{-3})^2}\right)^2 + (64\times10^6)^2\right]^{\frac{1}{2}} = \frac{300\times10^6}{1.2}$$

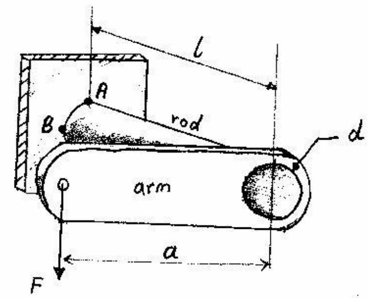
solve for P

P = 421.653 KN   Ans.

Ex. Determine the safety factors for the bracket rod based on both the distortion energy theory and max. shear theory

Given: $\sigma_y = 47000$ Psi, $l = 6$ in, $a = 8$ in, $d = 1.5$ in, $F = 1000$ lb and material is ductile.

▶ Point A (bending moment + torsion)



$$\sigma_x = \frac{My}{I} = \frac{(Fl)y}{I} = \frac{32(Fl)}{\pi d^3} = \frac{(32)(1000)(6)}{\pi(1.5)^3} = 18108 \text{ psi}$$

$$\tau_{xy} = \frac{Tr}{J} = \frac{16T}{\pi d^3} = \frac{16(Fa)}{\pi d^3} = 12072 \text{ psi}$$

$$\sigma_1 = \frac{\sigma_x+\sigma_y}{2} + \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}} = 24144 \text{ psi}$$

$$\sigma_2 = \frac{\sigma_x+\sigma_y}{2} - \left[\left(\frac{\sigma_x-\sigma_y}{2}\right)^2 + \tau_{xy}^2\right]^{\frac{1}{2}} = -6036 \text{ psi}$$

$$\bar{\sigma} = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{\frac{1}{2}} = \left[(24144)^2 - (24144)(-6036) + (-6036)^2\right]^{\frac{1}{2}} = 27661 \text{ Psi}$$

According to distortion energy (von-Mises) criterion

$$N = \frac{\sigma_y}{\bar{\sigma}} = \frac{47000}{27661} = 1.7$$

According to max. shear theory (Tresca)

$$N = \frac{\sigma_y}{\sigma_1 - \sigma_2} = \frac{47000}{(24144)-(-6036)} = 1.6$$

▶ Point B (transverse shear stress (comes from bending) + torsion)

$$\tau_{bending} = \frac{4}{3}\frac{V}{A} = \left(\frac{4}{3}\right)\frac{1000}{\frac{\pi}{4}(1.5)^2} = 755 \text{ psi} \quad , \quad \tau_{torsion} = \frac{16T}{\pi d^3} = 12072 \text{ psi}$$

$$\Rightarrow \tau_{max} = \tau_{bending} + \tau_{torsion} = 12827 \text{ psi}$$

According to distortion energy

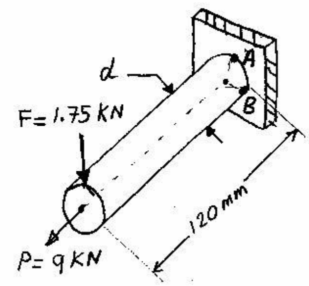
$$N = \frac{0.577\sigma_y}{\tau_{max}} = 2.1$$

According to max. shear theory

$$N = \frac{0.5\sigma_y}{\tau_{max}} = 1.8$$

Since the safety factor at point A (For both theories) is smaller than the safety factor at point B, thus point A will yield (Failure) First

Ex. A circular cantilever rod, shown in the Fig. below, is to be made from a ductile material with yield strength of 276 MPa. Using design factor $N = 2$, Find the appropriate diameter (d).



► Point A (normal stress + bending)

$$\sigma_x = \frac{P}{A} + \frac{My}{I}$$
$$= \frac{9\times10^3}{(\pi/4)d^2} + \frac{32(1.75\times10^3\times120\times10^{-3})}{\pi d^3} \quad \cdots ⓐ$$

$$\sigma_{1,2} = \frac{\sigma_x + \cancelto{0}{\sigma_y}}{2} \mp \left[\left(\frac{\sigma_x - \cancelto{0}{\sigma_y}}{2}\right)^2 + \cancelto{0}{\tau_{xy}^2}\right]^{\frac{1}{2}}$$

$$\sigma_1 = \sigma_x, \quad \sigma_2 = 0$$

Using von-Mises criterion

$$\bar{\sigma} = (\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2)^{\frac{1}{2}} = \sigma_1 = \sigma_x$$

$$\bar{\sigma} = \frac{\sigma_y}{N} = \frac{276\times10^6}{2} = 138\times10^6 \ \text{Pa} \quad \cdots ⓑ$$

From Eq. ⓐ and ⓑ

$$\frac{9\times10^3}{(\pi/4)d^2} + \frac{32(1.75\times10^3\times120\times10^{-3})}{\pi d^3} = 138\times10^6$$

or

$$138\times10^6 d^3 - 11459.156 d - 2139.04 = 0$$

solve for d $\qquad d = 26.04\times10^{-3}$ m

► Point B (normal stress + transverse shear)

$$\tau_{xy} = \left(\frac{4}{3}\right)\frac{F}{A} = \left(\frac{4}{3}\right)\frac{1.75\times10^3}{(\pi/4)d^2} = \frac{2970.89}{d^2}$$

$$\sigma_x = \frac{P}{A} = \frac{9\times10^3}{(\pi/4)d^2} = \frac{11459.156}{d^2}$$

$$(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2)^{\frac{1}{2}} = \frac{\sigma_y}{N}$$

$$\left[\left(\frac{11459.156}{d^2}\right)^2 + 3\left(\frac{2970.89}{d^2}\right)^2\right]^{\frac{1}{2}} = \frac{276\times10^6}{2} \Rightarrow d = 9.54\times10^{-3} \text{ m}$$

 Take the largest diameter, i.e $d = 26.04\times10^{-3}$ m Ans.