Design for High-cycle Fatigue ($N > 10^3$)

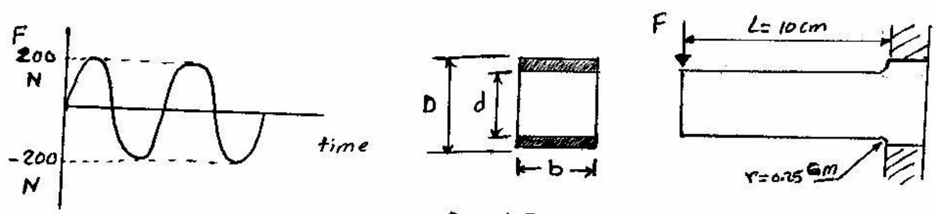
A) For uniaxial stress (Fully reversed)

$$N_f = \frac{S_n}{\bar{\sigma}_a} \qquad \text{safety factor}$$

if $N_f \geq 1$ safe design

$N < 1$ unsafe design

Ex. The cantilever beam shown ~~is~~, made ~~of~~ from steel with $\sigma_{ut} = 552$ MPa, is subjected to fully reversed load. Neglect shear stress effect, Estimate wheather the beam is safe or not safe at $N = 10^9$ cycles. The beam is machined surface and the operating temp. is $100°C$.



D = 1.3 cm
d = 1 cm
b = 1 cm

at the root of the cantilever, the bending moment is max.

$$M_a = F_a * L = 200 \times (10 \times 10^{-2}) = 20 \text{ N.m}$$

$$\sigma_{a\,nom.} = \frac{My}{I} = \frac{6M_a}{bd^2} = \frac{6(20)}{(1\times10^{-2})(1\times10^{-2})^2} = 120 \text{ MPa}$$

$$\frac{D}{d} = \frac{1.3}{1} = 1.3 \quad , \quad \frac{r}{d} = \frac{0.25}{1} = 0.25$$

From Fig. E-10

$A = 0.9588$, $b = -0.27269$

$$K_t \cong A\left(\frac{r}{d}\right)^b = 0.9588\,(0.25)^{-0.27269} = 1.4$$

From Fig. 6.36

For $r = 0.25$ cm $= 2.5$ mm and $\sigma_{ut} = 552$ MPa

$q = 0.8$

$\Rightarrow K_f = 1 + q(K_t - 1) = 1 + 0.8(1.4 - 1) = 1.32$

$\sigma_a = K_f \, \sigma_{a\,nom} = 1.32 \times 120 = 158.4 \text{ MPa}$

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \mp \left[\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 \right]^{\frac{1}{2}}$, $\tau_{xy} = 0$, $\sigma_y = 0$

$\sigma_1 = 158.4 \text{ MPa}$

$\sigma_2 = 0$

According to von-Mises criterion, the effective stress is

$\bar{\sigma}_a = \left(\sigma_1^2 - \sigma_1 \sigma_2 + \sigma_2^2 \right)_a^{\frac{1}{2}} = 158.4 \text{ MPa}$

$S_e' = 0.5 \, \sigma_{ut} = 0.5 \times 552 = 276 \text{ MPa}$

$C_{load} = 1$ (bending)

$A_{95} = 0.05 \underbrace{(d \times b)}_{\text{in (mm)}} = 0.05 \, (10 \times 10) = 5 \text{ mm}^2$

$d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}} = 8.08 \text{ mm}$

$C_{size} = 1.189 \left(d_{equiv} \right)^{-0.097} = 0.97$

$C_{Temp} = 1$

for machined surface Table 6-3 $A = 4.51$, $b = -0.265$

$C_{surface} \cong A \, (\underset{552}{\sigma_{ut}})^b = 0.846$ (Note σ_{ut} in MPa)

$C_{reliab} = 0.753$ for 99.9 %

$S_e = C_{load} \; C_{size} \; C_{surf} \; C_{Temp} \; C_{reliab} \; S_e'$

$= (1) \; (0.97) \; (0.846) \; (1) \; (0.753) \; 276 = 170.547 \text{ MPa}$

for $N = 10^9$ cycles $S_n = S_e$

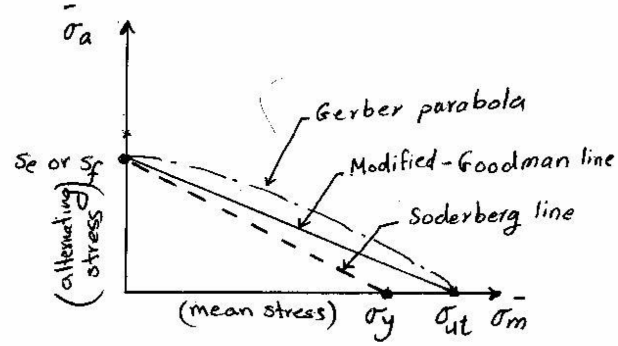
$\therefore N_f = \frac{S_n}{\bar{\sigma}_a} = \frac{170.547}{158.4} = 1.076 > 1$ safe

B) For fluctuating uniaxial stresses

Repeating and fluctuating stresses, have non-zero mean components ($\sigma_m \neq 0$) and these must be taken into account when determining the safety factor.

There are three lines plotted on $\bar{\sigma}_m - \bar{\sigma}_a$ axes represents the failure lines that modeld the fluctuating and repeating stresses:

1. Modified-Goodman line.
2. Gerber parabola.
3. Soderberg line.



Failure lines for fluctuating stresses

Modified-Goodman line:

$$\bar{\sigma}_a = S_e\left(1 - \frac{\bar{\sigma}_m}{\sigma_{ut}}\right) \quad \text{or} \quad \frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_{ut}} = 1$$

Gerber parabola:

$$\bar{\sigma}_a = S_e\left(1 - \frac{\bar{\sigma}_m^2}{\sigma_{ut}^2}\right) \quad \text{or} \quad \frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m^2}{\sigma_{ut}^2} = 1$$

Soderberg line:

$$\bar{\sigma}_a = S_e\left(1 - \frac{\bar{\sigma}_m}{\sigma_y}\right) \quad \text{or} \quad \frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_y} = 1$$

$\leftarrow$ (A)

$\bar{\sigma}_a$: Effective alternating stress component

$\bar{\sigma}_m$: Effective mean stress component

Applying stress - concentration Effects with Fluctuating stresses

For Fluctuating stress, two components of stress is found (σ_a) and (σ_m). The alternating component (σ_a) is treated as the same way as it was for the case of fully reversed stress (see Ex. in page 24). The mean component of stress (σ_m) is treated differently depending on the ductility or britteness of the material.

- if brittle:

$$\sigma_m = K \, \sigma_{m_{nom}}$$

- if ductile:

by using Dowling suggestion one of the three approaches may used

$$\text{if} \quad K_f \, |\sigma_{max}| < \sigma_y \longrightarrow K_{fm} = K_f$$

$$\text{if} \quad K_f \, |\sigma_{max}| > \sigma_y \longrightarrow K_{fm} = \frac{\sigma_y - K_f \sigma_a}{|\sigma_m|}$$

$$\text{if} \quad K_f \, |\sigma_{max} - \sigma_{min}| > 2\sigma_y \longrightarrow K_{fm} = 0$$

Determination of safety factor with fluctuating stresses

to find the safety factor N_f, replace σ_a by $N_f \sigma_a$ and σ_m by $N_f \sigma_m$ in Eq. (A). Thus we get the following:

modified - Goodman:
$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_{ut}} = \frac{1}{N_f}$$

Gerber parabola:
$$\frac{N_f \bar{\sigma}_a}{S_e} + \left(\frac{N_f \bar{\sigma}_m}{\sigma_{ut}}\right)^2 = 1$$

Soderberg:
$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_y} = \frac{1}{N_f}$$