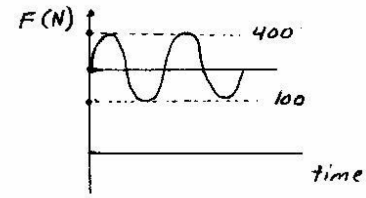
Ex. Repeat Ex. in page (24), with Fluctuating load as shown below. $\sigma_y = 462\ MPa, \sigma_{ut} = 552\ MPa$. Find the safety factor (N_f) using Modified-Goodman, Gerber, and soderberg criterias



$$F_m = \frac{F_{max} + F_{min}}{2} = \frac{400 + 100}{2} = 250\ N$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{400 - 100}{2} = 150\ N$$

$F_{max} = 400\ N,\ F_{min} = 100\ N$

$$M_m = F_m * L = 250 * (10 \times 10^{-2}) = 25\ N.m$$

$$M_a = F_a * L = 150 * (10 \times 10^{-2}) = 15\ N.m$$

$$M_{max} = F_{max} * L = 400 * (10 \times 10^{-2}) = 40\ N.m$$

$$M_{min} = F_{min} * L = 100 * (10 \times 10^{-2}) = 10\ N.m$$

$K_t = 1.4$ and $K_f = 1.32$ (as found before)

$$\sigma_{a\,nom} = \frac{M_a y}{I} = \frac{6\,M_a}{b d^2} \text{ (For rectangle shape) } = \frac{6 * 15}{(1\times10^{-2})(1\times10^{-2})^2} = 90\ MPa$$

$$\sigma_{m\,nom} = \frac{M_m y}{I} = \frac{6\,M_m}{b d^2} = \frac{6 * 25}{(1\times10^{-2})(1\times10^{-2})^2} = 150\ MPa$$

$$\sigma_{max} = \frac{6 M_{max}}{b d^2} = 240\ MPa\ , \quad \sigma_{min} = \frac{6\,M_{min}}{b\,d^2} = 60\ MPa$$

since

$$K_f\ |\sigma_{max}| < \sigma_y = 462 \quad \Rightarrow \quad K_{fm} = K_f = 1.32$$

$$\sigma_a = K_f\ \sigma_{a\,nom} = 1.32 \times 90 = 118.8\ MPa$$

$$\sigma_m = K_{fm}\ \sigma_{m\,nom} = 1.32 \times 150 = 198\ MPa$$

$$\sigma_{1,2}\Big)_a = \frac{\sigma_x + \cancel{\sigma_y}^{\,0}}{2}\Big)_a \mp \left[\left(\frac{\sigma_x - \cancel{\sigma_y}^{\,0}}{2}\right)^2 + \cancel{\tau_{xy}^2}^{\,0}\right]^{\frac{1}{2}}_a$$

$$\left.\begin{array}{l} \sigma_1)_a = 118.8 \\ \sigma_2)_a = 0 \end{array}\right\} \ \vec{\sigma_a} = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)^{\frac{1}{2}}_a = 118.8\ MPa$$

$$\sigma_{1,2})_m = \frac{\sigma_x + \sigma_y}{2}\Big)_m \mp \left[\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2\right]_m^{\frac{1}{2}}$$

$$\left.\begin{array}{l} \sigma_1)_m = 198 \text{ MPa} \\ \sigma_2)_m = 0 \end{array}\right\} \quad \bar{\sigma}_m = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)_m^{\frac{1}{2}} = 198 \text{ MPa}$$

$S_e = 170.547$ MPa (as found before)

- Using modified - Goodman

$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_{ut}} = \frac{1}{N_f} \quad , \quad \frac{118.8}{170.547} + \frac{198}{552} = \frac{1}{N_f} \Rightarrow N_f = 0.947 < 1$$

unsafe

- Gerber parabola

$$\frac{N_f\,\bar{\sigma}_a}{S_e} + \left(\frac{N_f\,\bar{\sigma}_m}{\sigma_{ut}}\right)^2 = 1 \,,$$

$$0.1286\, N_f^2 + 0.6965\, N_f - 1 = 0 \qquad \text{(second degree eq.)}$$

$$(N_f)_{1,2} = \frac{-0.6965 \mp \left(0.6965^2 - 4 * 0.1286 * (-1)\right)^{\frac{1}{2}}}{2 \times 0.1286}$$

$$N_{f_1} = 1.179 > 1 \quad \text{safe}$$

$$N_{f_2} = -6.595 \quad \text{(neglected} < 0)$$

- Soderberg

$$\frac{\bar{\sigma}_a}{S_e} + \frac{\bar{\sigma}_m}{\sigma_y} = \frac{1}{N_f} \quad , \quad \frac{118.8}{170.547} + \frac{198}{462} = \frac{1}{N_f} \Rightarrow N_f = 0.888 < 1$$

unsafe

The results of (N_f) shows that soderberg criterion is more conservative than other critirions

C) For Fully Reversed Multiaxial Stresses

$$\bar{\sigma}_m = 0$$

$$\bar{\sigma}_a = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)_a^{\frac{1}{2}}$$

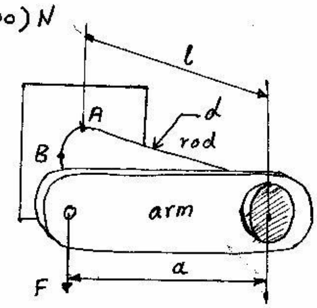
$$N_f = \frac{S_n}{\bar{\sigma}_a}$$

D) For Fluctuating Multiaxial Stresses

$$\bar{\sigma}_a = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)_a^{\frac{1}{2}} = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2\right)_a^{\frac{1}{2}}$$

$$\bar{\sigma}_m = \left(\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2\right)_m^{\frac{1}{2}} = \left(\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}\right)_m^{\frac{1}{2}}$$

Ex. Determine the safety factor for the bracket rod shown below for $N = 6\times10^7$ cycles. The notch radius at the wall is $r = 0.25$ cm. the braket is made from Aluminum 2024, cold-drawn with $\sigma_y = 290$ MPa, $\sigma_{ut} = 441$ MPa. $L = 10$ cm, $a = 8$ cm, $d = 2$ cm.
the applied load F is varied from (340) to (-200) N sinusoidally. take $K_t = 1.7$, $K_{ts} = 1.35$.



► since $\sigma_{ut} = 441$ MPa

$\therefore S_f' = 130$ MPa (at 5×10^8 cycles)

$C_{load} = 1$ (bending)

From Fig 6-25

$$d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}}$$

for non-rotating solid rod

$A_{95} = 0.010462\, d^2 = 4.1848\ mm^2$

$\Rightarrow d_{equiv} = 7.391$ mm < 8 mm $\Rightarrow C_{size} = 1$

$C_{surf} \cong A(\sigma_{ut})^b = 4.51\,(441)^{-0.265} = 0.898$ (for cold-drawn)

$C_{temp} = 1$

$C_{reliab} = 0.753$ for 99.9 % reliability

$$S_f = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_f'$$
$$= (1)\ (1)\ (0.898)\ (1)\ (0.753)\ 130 = 87.905\ MPa$$

$S_o = 0.9\ \sigma_{ut}$ for bending

$$S_o = 0.9 \times 441 = 396.9\ MPa$$

$$b = -\frac{1}{5.699} \log\left(\frac{S_o}{S_f}\right) = -\frac{1}{5.699} \log\left(\frac{396.9}{87.905}\right) = -0.1148$$

$$\log(a) = \log(S_o) - 3b = \log(396.9 \times 10^6) - 3(-0.1148)$$

$$a = 877.164\ MPa$$

$$S_n = a N^b = 877.164 \times 10^6 \,(6 \times 10^7)^{-0.1148} = 112.24\ MPa$$

From Fig 6-36 (heat-treated Aluminum)

for $r = 0.25\ cm = 2.5\ mm$ and $\sigma_{ut} = 441\ MPa$

$$\Rightarrow q \equiv 0.665$$

$$k_f = 1 + q(K_t - 1) = 1 + 0.665\,(1.7 - 1) = 1.4655$$

$$K_{fs} = 1 + q\,(K_{ts} - 1) = 1 + 0.665\,(1.35 - 1) = 1.2327$$

$$F_a = \frac{F_{max} - F_{min}}{2} = \frac{340 - (-200)}{2} = 270\ N$$

$$F_m = \frac{F_{max} - F_{min}}{2} = \frac{340 + (-200)}{2} = 70\ N$$

$$M_a = F_a * L = 270 * (10 \times 10^{-2}) = 27\ N.m$$

$$M_m = F_m * L = 70 * (10 \times 10^{-2}) = 7\ N.m$$

$$T_a = F_a * a = 270 * (8 \times 10^{-2}) = 21.6\ N.m$$

$$T_m = F_m * a = 70 * (8 \times 10^{-2}) = 5.6\ N.m$$

Since $K_f\, |\sigma_{max}| < \sigma_y$ where $\sigma_{max} = \frac{32\, M_{max}}{\pi d^3} = \frac{32\, [\overset{F_{max}}{\overbrace{340} } * (10 \times 10^{-2})]}{\pi (2 \times 10^{-2})^3}$

$1.4655\, |43.44\ MPa| < 290\ MPa$ $\qquad = 43.29\ MPa$

$$\Rightarrow K_{fm} = K_f = 1.4655\ ,\quad K_{fsm} = K_{fs} = 1.2327$$

31

$$\sigma_a\ nom = \frac{32\ M_a}{\pi d^3} = 34.377\ MPa$$

$$\sigma_m\ nom = \frac{32\ M_m}{\pi d^3} = 8.9126\ MPa$$

$$\tau_{a\ nom})_{torsion} = \frac{16\ T_a}{\pi d^3} = 13.751\ MPa$$

$$\tau_{m\ nom})_{torsion} = \frac{16\ T_m}{\pi d^3} = 3.565\ MPa$$

$$\tau_{a\ nom})_{transverse} = \frac{4}{3}\ \frac{F_a}{A} = \frac{4}{3} \times \frac{270}{\frac{\pi}{4}(0.02)^2} = 1.1459\ MPa$$

$$\tau_{m\ nom})_{Transverse} = \frac{4}{3}\ \frac{F_m}{A} = 0.29709\ MPa$$

$$\Rightarrow\ \sigma_a = K_f\ \sigma_{a\ nom} = (1.4655)(34.377) = 50.3795\ MPa$$

$$\sigma_m = K_{fm}\ \sigma_{m\ nom} = (1.4655)(8.9126) = 13.0614\ MPa$$

$$\tau_a)_{torsion} = K_{fs}\ \tau_{a\ nom})_{torsion} = (1.2327)(13.751) = 16.95\ MPa$$

$$\tau_m)_{torsion} = K_{fsm}\ \tau_{m\ nom})_{torsion} = (1.2327)(3.565) = 4.3945\ MPa$$

$$\tau_a)_{transverse} = K_{fs}\ \tau_{a\ nom})_{transverse} = (1.2327)(1.1459) = 1.4125\ MPa$$

$$\tau_m)_{transverse} = K_{fsm}\ \tau_{m\ nom})_{transverse} = (1.2327)(0.29709) = 0.3662\ MPa$$

At point (A) (bending + torsion)

$$\bar{\sigma}_a = (\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2)_a^{\frac{1}{2}} = \left[(50.3795)^2 + 0 - 0 + 3*(16.95)^2\right]^{\frac{1}{2}}$$

$$= 58.309\ MPa$$

$$\bar{\sigma}_m = (\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2)_m^{\frac{1}{2}}$$

$$= \left[(13.0614)^2 + 0 - 0 + 3*(4.3945)^2\right]^{\frac{1}{2}} = 15.1173\ MPa$$

Using Modified-Goodman

$$\frac{\bar{\sigma}_a}{S_{f}=S_n} + \frac{\bar{\sigma}_m}{\sigma_{ut}} = \frac{1}{N_f} \Rightarrow \frac{58.309}{112.24} + \frac{15.1173}{441} = \frac{1}{N_f} \Rightarrow N_f = 1.805 > 1$$

Point (B) ($\tau_{torsion} + \tau_{transverse}$)

$$\tau_a)_{total} = \tau_a)_{torsion} + \tau_a)_{transverse}$$

$$= 16.95 + 1.4125 = 18.3625 \text{ MPa}$$

$$\tau_m)_{total} = \tau_m)_{torsion} + \tau_m)_{transverse}$$

$$= 4.3945 + 0.3662 = 4.7607 \text{ MPa}$$

$$\bar{\sigma}_a = \left(\cancel{\sigma_x^2} + \cancel{\sigma_y^2} - \cancel{\sigma_x \sigma_y} + 3\tau_{xy}^2\right)_a^{\frac{1}{2}} = (3 \times 18.3625)^{\frac{1}{2}} = 7.422 \text{ MPa}$$

$$\bar{\sigma}_m = \left(\cancel{\sigma_x^2} + \cancel{\sigma_y^2} - \cancel{\sigma_x \sigma_y} + 3\tau_{xy}^2\right)_m^{\frac{1}{2}} = (3 \times 4.7607)^{\frac{1}{2}} = 3.779 \text{ MPa}$$

$$\frac{\bar{\sigma}_a}{s_f = s_n} + \frac{\bar{\sigma}_m}{\sigma_{ut}} = \frac{1}{N_f}$$

$$\frac{7.422}{112.24} + \frac{3.779}{441} = \frac{1}{N_f}$$

$$\Rightarrow N_f = 13.387 \quad > 1 \quad \text{safe}$$

Both points (A) and (B) are safe against fatigue failure