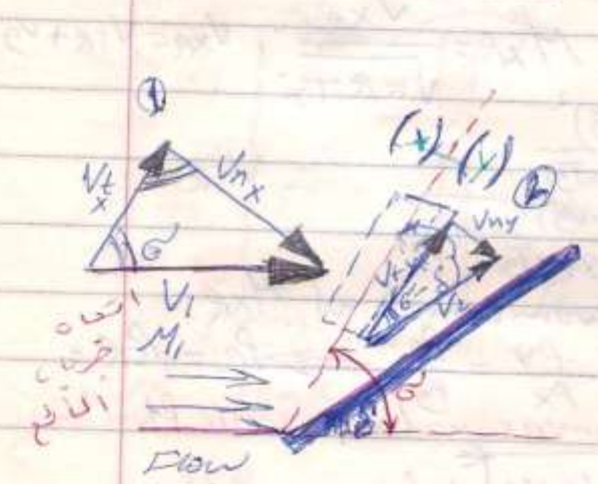
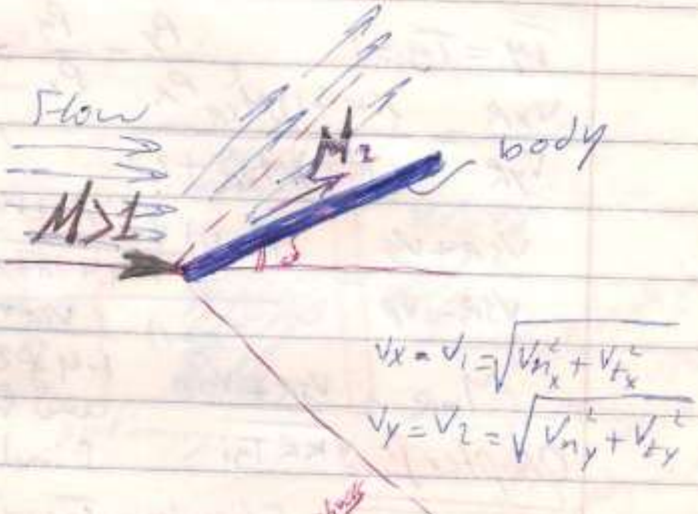
Oblique Shock Wave

When a uniform supersonic (جبر) is forced to change its direction due to the presence (وجود) of a body in the flow (اتجاه الجريان), then the stream adjust to the presence of the body gradually (يستطيع تغيير اتجاهه), such an adjustment is only possible by a sudden change in flow properties with the aid (بمساعدة) of an oblique shock wave.

A simple case in supersonic flow at a two-dimensional concave corner as shown in fig. For small deflection angles, the flow adjusts to the presence of such a corner by means of an oblique shock wave, which is attached to the corner. The flow after the shock wave is uniform (منتظم) and parallel to the wall (السطح), with the entire flow having been turned through the defflection (مستمرة) angle δ

Governing Equations





$$V_x = V_1 = \sqrt{V_{n_x}^2 + V_{t_x}^2}$$
$$V_y = V_2 = \sqrt{V_{n_y}^2 + V_{t_y}^2}$$

زاوية انحراف السطح = δ

زاوية الموجة = σ

V_{t_x} = مركبة السرعة الموازية للصدمة

V_{n_x} = = العمودية = =

* خواص الساكنة ثابتة لانها لا تتأثر بالسرعة مثلاً $T_n = T_t$ لذلك

$$P_x = P_1 \;,\; P_y = P_2$$
$$T_x = T_1 \;,\; T_y = T_2$$

(a) Continuity Equation:

* For steady and uniform flow upstream and downstream of the oblique shock wave, the mass flow rate is constant. $\dot{m} = \text{const.}$

لذلك سوف نفصل المعادلات (العمودية) لأن تأثير الصدمة على المركبة العمودية

$$\dot{m} = \rho_x V_{xn} A_x = \rho_y A_y V_{yn}$$

بسبب السمك المتناهي بالصغر

the shock wave is very thin so that $A_x = A_y$

$$\rho_x V_{xn} = \rho_y V_{yn}$$

b) Momentum Equations-

In the direction normal to the shock.

$$\Sigma F = \iint_s V(\rho V dA)$$

$$P_x A_x - P_y A_y = \rho_y A_y V_{yn}^2 - \rho_x A_x V_{xn}^2$$

or

$$P_x - P_y = \rho_y V_{yn}^2 - \rho_x V_{xn}^2$$

In the tangential direction there is no change in the pressure so that-

$$\dot{m} V_{xt} = V_{xt}(\rho_x A_x V_{xn}) = V_{yt}(\rho_y A_y V_{yn}) = \dot{m} V_{yt}$$

from eqn. (5-1)

$$\rho_x A_x V_{xn} = \rho_y A_y V_{yn}$$

so that

$$\boxed{V_{xt} = V_{yt}}$$

The first law of thermodynamics for adiabatic and steady flow, the first law of thermodynamics reduce to.

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$h_1 + \frac{(V_{xn}^2 + V_{xt}^2)}{2} = h_2 + \frac{(V_{yn}^2 + V_{yt}^2)}{2}$$

while

$$V_{ty} = V_{tx}$$

$$\boxed{h_1 + \frac{V_{xn}^2}{2} = h_2 + \frac{V_{yn}^2}{2}}$$

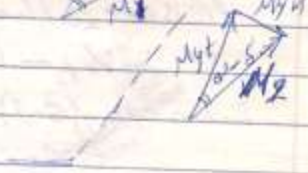
At this point one many observe that the equations is similler to that of normal shock. In other words, an oblique shock wave acts as a normal shock wave for the velocity normal to the flow, while the tangential velocity compenant remains **unchanged**

(1) $M_{x_n} = M_x = M_1 \sin\sigma$

(2) $M_{x_t} = M_1 \cos\sigma$

(3) $M_{y_n} = M_y = M_2 \sin(\sigma - \delta)$

(4) $M_{y_t} = M_2 \cos(\sigma - \delta)$



In most case, the wave angle is not known, but rather M_1 and δ appear as the independent variables, therefore it is more advantageous to be able to express the wave angle to σ and M_2 in term of M_1 and δ.

$$\tan\sigma = \frac{V_{x_n}}{V_{x_t}}$$

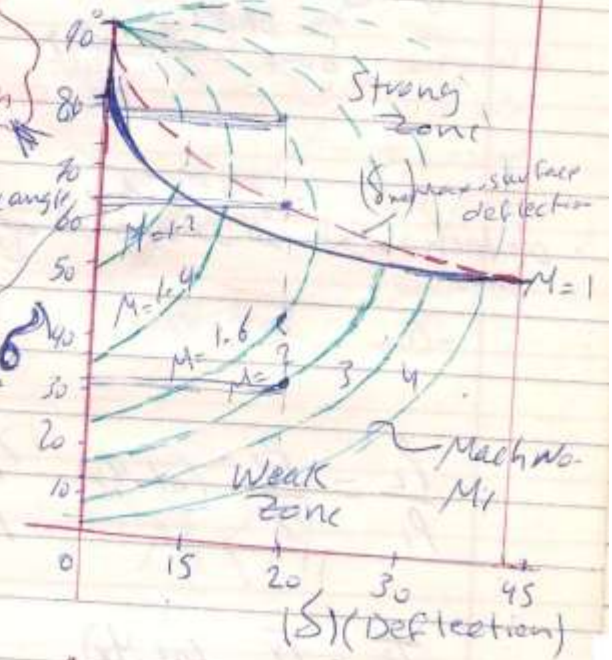
$$= \frac{(k-1) M_{x_n}^2 + 2}{(k+1) M_{x_n}^2} = \frac{V_y}{V_x} = \frac{\tan(\sigma - \delta)}{\tan\sigma}$$

$$\frac{\tan(\sigma - \delta)}{\tan\sigma} = \frac{(k-1)(M_1^2 \sin^2\sigma) + 2}{(k+1)(M_1^2 \sin^2\sigma)}$$

$$\tan\delta = 2 \cot\sigma \frac{M_1^2 \sin^2\sigma - 1}{M_1^2 (k + \cos 2\sigma) + 2}$$

$$\cot\delta = \left(\frac{k+1}{2}\cdot\frac{M_1^2}{M_1^2\sin^2\sigma - 1} - 1\right)\tan\sigma$$

Characteristics of the Oblique shock Equation



لايوجد حل واحد فقط

② M_1 الحل للمعادلة لها حلين احدهما strong soln وهو الحل الناتج في strong zone والآخر weak soln والناتج في weak zone.

③ قيمة P_B هي التي تحدد نوع الحل اذا كان strong او weak

اذا كان P_B قليل بالنسبة P_{atm} فنختار الحل weak soln

اما اذا كان P_B عالي بالنسبة P_{atm} فنختار الحل strong

① لعدد ماخ M_1 و δ معينة [illegible]

الحل ② M_1 [illegible] له

② اقوى oblique shock هو N.shock

لذلك فان الـ obli. shock الاقوى

الـ N-shock تسمى strong obli.

* لكل سرعة (M) معينة توجد زاوية حرجة تكون عندها oblique shock متماسة مع الجسم أي ملتصقة.

* متى يحدث انفصال لا توجد قيمة لـ δ (عند سرعة عالية) وتتحول هذه الموجة detach shock
وتقع أمام الجسم.

$$\frac{P_2}{P_1} = \frac{P_y}{P_x} \quad (\text{at } M_x) \qquad P_2 = P_y, \quad P_1 = P_x$$

$$\frac{T_2}{T_1} = \frac{T_y}{T_x} \quad (\text{at } M_x) \qquad \frac{\rho_2}{\rho_1} = \frac{\rho_y}{\rho_x}$$

$P_{01} \neq P_{0x}$ $\qquad P_{01} \xrightarrow{at} V_1^2 = V_{xn}^2 + V_{xt}^2$ or at $M_1 \xrightarrow{A_1}$

$P_{02} \cancel{\neq} P_{0y}$ $\qquad P_{0x} \xrightarrow{at} (V_{xn}^2)$ or at $M_{xn} \xrightarrow{A_2}$

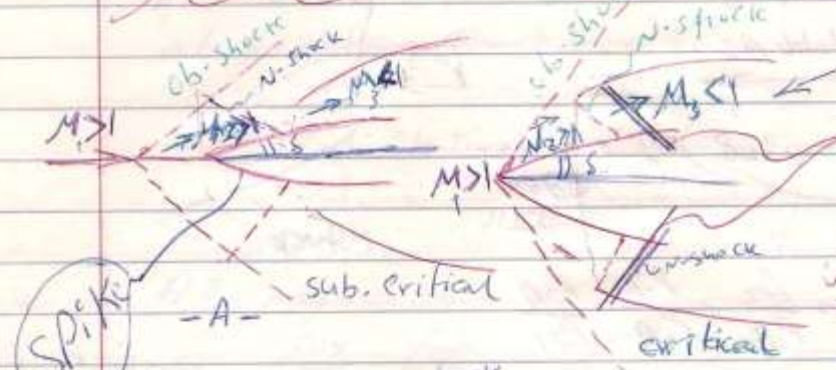
$$\frac{P_{02}}{P_{01}} = \frac{P_{0y}}{P_{0x}} \qquad \boxed{V_{t1} = V_{t2}}$$

at $M_1 \xrightarrow{A_1} \frac{P_1}{P_{01}} \Rightarrow P_{01}$ ~~at M_1~~

at $M_{1n} = M_x$ (A3) $\Rightarrow \frac{P_{02}}{P_{01}} = \frac{P_{0y}}{P_{0x}} = \checkmark$

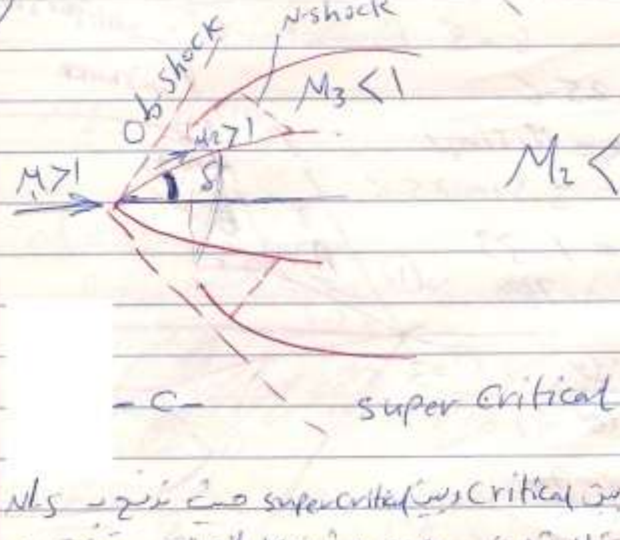
$$\Rightarrow P_{02} = \checkmark$$

Intake Diffuser of supersonic



Best Case

* تصبح الـ shock داخل المدخل فيحصل خسارة قليلة في الكتلة ولكن نحصل على حالة مستقرة



$M_2 < M_1$

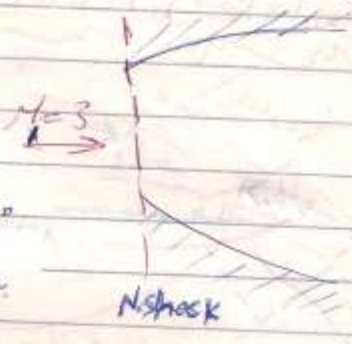
* الحالة المثالية هي خليط بين critical وبين supercritical حيث تزحزح الـ N/S
إلى داخل الـ Intake بمسافة قليلة أقل من super critical حيث نحصل على حالة مستقرة مع
خسارة قليلة بالكتلة الداخلة.

EX: 6.3 P. 130

at $M=3$

From table A3

$$\frac{P_{oy}}{P_{ox}} = 0.328$$



$$\frac{P_{o2}}{P_{o1}} = \frac{P_{o2}}{P_{ob}} \cdot \frac{P_{ob}}{P_{oA}} \cdot \frac{P_{oA}}{P_{o1}}$$

$M = 3$ $\delta = 8°$ From chart

$\sin\theta = 25.6$

$M_{x_1} = M_1 \sin\theta$

$= 3 \sin 25.6$

$= 1.33$

table A3

$M_y = 0.784$

$$M_A = \frac{M_y}{\sin(\theta - \delta)}$$

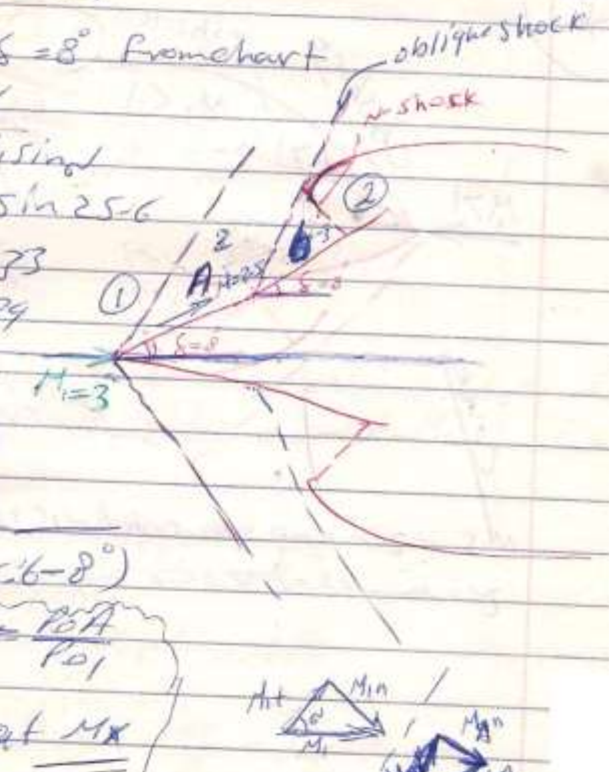
$$M_A = \frac{0.78}{\sin(25.6 - 8°)}$$

$$\frac{P_{oy}}{P_{ox}} = 0.9794 = \frac{P_{oA}}{P_{o1}}$$

$$\frac{P_{o2}}{P_{o1}} = \frac{P_{oy}}{P_{ox}} \text{ at } M_x$$

$P_{o2} \neq P_{oy}$

$P_{o1} \neq P_{ox}$



$M_A = 2.5$

at $\delta = 8^\circ$ from chart find σ_2'

$\sigma_2' = 29^\circ$

$M_x = M_A \sin\sigma$

$= 2.5 \sin(29)$

$M_x = 1.2$

$\xrightarrow{A3} M_y = 0.89$

$\frac{P_{0y}}{P_{0x}} = \frac{P_{0b}}{P_{0A}} = 0.994$

$M_b = \frac{0.84}{\sin(29.8)}$

$M_b = 2.3$

from $\xrightarrow{table\ A3}$ at $M_b = M_x = 2.3$

$\frac{P_{02}}{P_{0b}} = 0.5280$

$\frac{P_{02}}{P_{01}} = 0.582 \times 0.99 \times 0.979$

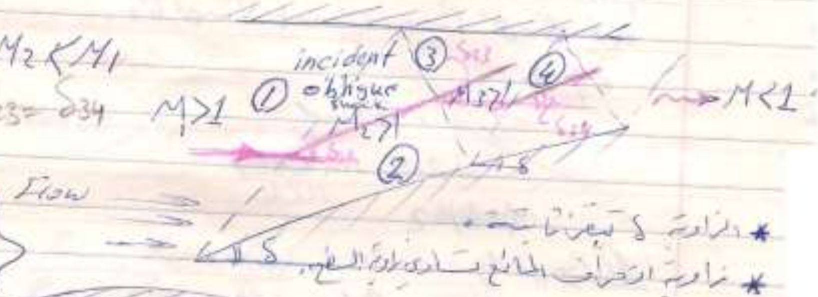
$= 0.56 \Rightarrow 44\%$ الخسارة

قلت

Reflection and intraction of obliqueshocks:-

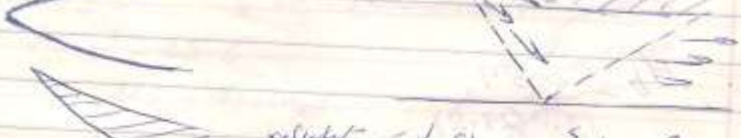
$M_3 < M_2 < M_1$

$\delta_{12} = \delta_{23} = \delta_{34}$



$\delta_{12} = \delta_{23} = \delta_{34}$

* زاوية δ تبعا ...
* زاوية انحراف المائع تساوي زاوية السطح.



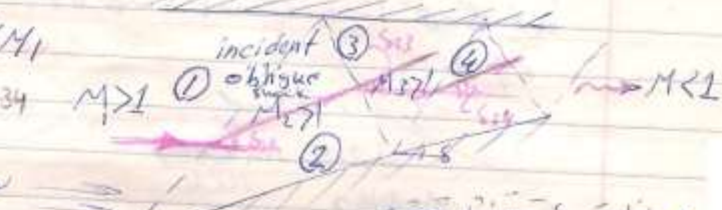
* إذا كان المائع Strong لا يحدث reflected
لأن المائع يتحول مباشرة إلى sub.

When a weak, two-dimensional Compression wave is reflected from a straight solid boundary, a wave of the same family is produced in order to maintain the flow parallel to the wall, The reflected compression wave is weaker than the incident wave, because its upstream number is less than that of the incident wave. Fig. show the reflection of shock wave. in region (1) the flow is parallel to the upper wall but turns through an angle δ_{12} toward the shock to assume a direction parallel to the lower wall in region (2).

Reflection and intraction of oblique shocks

$M_3 < M_2 < M_1$

$\delta_{12} = \delta_{23} = \delta_{34}$



$\boxed{\delta_{12} = \delta_{23} = \delta_{34}}$

* زاوية δ تبقى ثابتة

* زاوية انحراف المائع تساوي زاوية الـ δ

* اذا كان المائع strong reflected لصدمة

المائع يتحول مباشرة الى sub-

When a weak, two-dimensional compression wave is reflected from a straight solid boundary, a wave of the same family is produced in order to maintain the flow parallel to the wall. The reflected compression wave is weaker than the incident wave, because its upstream number is less than that of the incident wave. Fig. show the reflection of shock wave. In region (1) the flow is parallel to the upper wall but turns through an angle δ_{12} toward the shock to assume a direction parallel to the lower wall in region (2).

In region (3), the flow turns through an angle ($\delta_{23} = \delta_{12}$) to assume once again a direction parallel to the upper wall. Thus the streamlines experience equal but opposite turning angles as they cross the incident and reflected wave. The streamlines in region 1 and region 3 point the same direction.

Note that:-

$$M_1 > M_2 \qquad \delta_{12} = \delta_{23} = \delta_{34}$$

$$M_2 > M_3$$

so that as the Mach number downstream decrease the shock strength become weaker and weaker until the flow will be subsonic.-

Exe the angle of incident of a compression wave in air (40°) with respect to that wall as shown of the upstream of the incident wave is 2.2 determine the (M) upstream and down stream of the reflected wave with respect to the uper wall.

Sol

$M_1 = 2.2$
$\sigma_1 = 40^\circ$ $\Rightarrow$ From chart $\delta = 14^\circ$

$M_x = M_1 \sin\sigma$
$= 2.2 \sin 40^\circ$
$= 1.414 \xrightarrow{\text{table A3}} M_y = 0.7355$

$$M_2 = \frac{M_y}{\sin(\sigma' - \delta)} = \frac{0.7355}{\sin(40-14)} = 1.6778$$

From chart at $M_2 = 1.59$ 1.667 and $\delta = 14$ find σ

$\sigma'_{23} = 59^\circ$ $\sigma_{23} = 55$

$\therefore M_x = M_2 \sin\sigma$
$= 1.59 \sin 59$ 1.667
$= 1.36$

from tables of N.S at M_x find $M_y = 0.757$

$$\therefore M_3 = \frac{M_y}{\sin(\sigma'_{23} - \delta)} = 1.163$$