Plane stress and plane strain Conditions:-

plane stress occurs in a thin plate while plane strain occurs in a thick plate. There are differences in the Crack Theory between The two Cases:-

- in a thin sheet, at the tip of Stressed crack, the thickness of the specimen decreases because of the poisson Contraction; plane stress Conditions occur at the crack tip.

- In the thick plate, at the tip of the stressed Crack, the thickness of the Specimen does not decrease by poisson Contraction: through thickness Stress are generated which off-set ~~poisson~~ poisson Contraction. This phenomenon is known as elastic Constraint; it is generated by the material surrounding the crack (outside the region of high stress at the crack tip, plane strain Conditions occur at the crack tip.

For plane stress Condition: $\sigma_F = \left(\frac{E G_c}{\pi a}\right)^{1/2}$

a: half or Crack length ; G_c is the fracture energy ;

The more useful parameter in the plane stress is Critical stress intensity factor (K_c) which is defined in a wide Sheet (infinite width) : $K_c = \sigma_F (\pi a)^{1/2}$

or $K_c = (E G_c)^{1/2}$

The determination of K_c is required to measure the value of σ_F at which a crack of length 2a in a thin wide plate begins to propagate.

The Use of K_c to determine whether or not a given thin sheet will fracture under a stress σ implies that the size of the largest Crack in the Sheet is known to the designer , The stress intensity factor

$K = \sigma (\pi a)^{1/2}$ Can be Computed and Compared with K_c ; the crack will not Spread for $K < K_c$.

Example 1:- A Sharp Central Crack of length 60 mm in a wide thin Sheet of a glassy plastic Commences to propagate at $\sigma_F = 3.26$ Mpa. (1) Find K_c; (2) Find G_c if you given that $E = 3$ Gpa; and (3) will a crack of length 2mm in a Similar Sheet fracture under $\sigma = 10$ Mpa?

(1) $K_c = \sigma_F (\pi a)^{1/2} = 3.26(\pi * \frac{1}{2} * 60 * 10^{-3})^{1/2}$

$= 1.00$ Mpa$\cdot m^{1/2}$ (or $m^{0.5}$)

(2) $G_c = K_c^2/E = (10^6)^2/3*10^9 = 333$ J/m^2

(3) $K = \sigma(\pi a)^{1/2} = 10(\pi * 1 * 10^{-3})^{1/2}$

$= 0.56$ Mpa $m^{0.5}$

K is 56% of K_c, The sheet will not fracture.

- The parameters which describe plane strain fracture are G_{Ic} and K_{Ic}; The Subscript I refers to the method of opening the Crack assumed here where there is two modes of fracture have been defined both involving shear on the Crack plane. In mode II, Shear is parallel to the crack propagation direction. In mode III

Shear is normal to the propagation direction. These types of fracture are important in long fiber composites and in adhesives because of anisotropy. They are of minor importance in thermoplastics. Cracks in isotropic materials tend to turn in a direction normal to the tensile stress, giving mode I fracture, whatever the initial orientation of the crack plane.

For a thick plate of infinite width containing a crack of length 2a, the fracture stress is:-

$$\sigma_F = \left(\frac{E G_{IC}}{\pi (1-\nu^2) a} \right)^{1/2} \quad \text{(plane strain)}$$

ν: poisson's ratio ; The stress intensity factor is:-

$$K_{IC} = \sigma_F (\pi a)^{1/2}$$

$$K_{IC} = \left(\frac{E G_C}{(1-\nu^2)} \right)^{1/2}$$

For the calculation of in-use fracture stresses, plane strain is normally the best assumption, because materials show minimum toughness in plane strain, so →

$G_{IC} < G_C$; and $K_{IC} < K_C$

(If a Component will not fracture in plane strain, it will Certainly not fracture in plane stress. The stress intensity factor in plane strain under stress σ for a wide plate Containing a crack of length 2a is :-

$$K_I = \sigma (\pi a)^{1/2}$$

For $K_I < K_{IC}$ the plate will not fracture.

Example 2 :- A thick wide plate of polystyrene Contains a Central sharp crack of length $2a = 40$ mm. The crack is found to propagate at $\sigma_F = 4.20$ Mpa. ① Find K_{IC} ② Find G_{IC} given that $E = 3$ Gpa and $\nu = 0.4$; and ③ will a crack of length 2mm in a similar sheet fracture if $\sigma = 10$ Mpa?

① $K_{IC} = \sigma_F (\pi a)^{1/2} = 4.2 (\pi * \frac{1}{2} * 40 * 10^{-3})^{1/2}$

$= 1.05 \text{ Mpa } m^{0.5}$

② $G_{IC} = \dfrac{(1-\nu^2)\, K_{IC}^2}{E} = \dfrac{(1-0.4^2)\,(1.053 * 10^6)^2}{3 * 10^9}$

$= 310 \ J/m^2$

③ $K_I = 10 * 10^6 (\pi * 1 * 10^{-3})^{1/2}$

$= 0.56 \text{ MPa } m^{0.5}$ → K_I is 53% of K_{IC}, The plate will not fracture