EXAMPLES :

* given $y = f(u)$ and $u = g(x)$, find $dy/dx = f'(g(x))g'(x)$

1. $y = 6u - 9, \quad u = (1/2)x^4$
2. $y = 2u^3, \quad u = 8x - 1$
3. $y = \sin u, \quad u = 3x + 1$
4. $y = \cos u, \quad u = -x/3$
5. $y = \cos u, \quad u = \sin x$
6. $y = \sin u, \quad u = x - \cos x$
7. $y = \tan u, \quad u = 10x - 5$
8. $y = -\sec u, \quad u = x^2 + 7x$

SOL :

1. $f(u) = 6u - 9 \Rightarrow f'(u) = 6 \Rightarrow f'(g(x)) = 6;\ g(x) = \frac{1}{2}x^4 \Rightarrow g'(x) = 2x^3$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
   $= 6 \cdot 2x^3 = 12x^3$

2. $f(u) = 2u^3 \Rightarrow f'(u) = 6u^2 \Rightarrow f'(g(x)) = 6(8x - 1)^2;\ g(x) = 8x - 1 \Rightarrow g'(x) = 8$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
   $= 6(8x - 1)^2 \cdot 8 = 48(8x - 1)^2$

3. $f(u) = \sin u \Rightarrow f'(u) = \cos u \Rightarrow f'(g(x)) = \cos(3x + 1);\ g(x) = 3x + 1 \Rightarrow g'(x) = 3$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
   $= (\cos(3x + 1))(3) = 3\cos(3x + 1)$

4. $f(u) = \cos u \Rightarrow f'(u) = -\sin u \Rightarrow f'(g(x)) = -\sin\left(\frac{-x}{3}\right);\ g(x) = \frac{-x}{3} \Rightarrow g'(x) = -\frac{1}{3}$; therefore $\frac{dy}{dx} = f'(g(x))g'(x)$
   $= -\sin\left(\frac{-x}{3}\right) \cdot \left(\frac{-1}{3}\right) = \frac{1}{3}\sin\left(\frac{-x}{3}\right)$

5. $f(u) = \cos u \Rightarrow f'(u) = -\sin u \Rightarrow f'(g(x)) = -\sin(\sin x);\ g(x) = \sin x \Rightarrow g'(x) = \cos x$; therefore
   $\frac{dy}{dx} = f'(g(x))g'(x) = -(\sin(\sin x))\cos x$

6. $f(u) = \sin u \Rightarrow f'(u) = \cos u \Rightarrow f'(g(x)) = \cos(x - \cos x);\ g(x) = x - \cos x \Rightarrow g'(x) = 1 + \sin x$; therefore
   $\frac{dy}{dx} = f'(g(x))g'(x) = (\cos(x - \cos x))(1 + \sin x)$

7. $f(u) = \tan u \Rightarrow f'(u) = \sec^2 u \Rightarrow f'(g(x)) = \sec^2(10x - 5);\ g(x) = 10x - 5 \Rightarrow g'(x) = 10$; therefore
   $\frac{dy}{dx} = f'(g(x))g'(x) = (\sec^2(10x - 5))(10) = 10\sec^2(10x - 5)$

8. $f(u) = -\sec u \Rightarrow f'(u) = -\sec u \tan u \Rightarrow f'(g(x)) = -\sec(x^2 + 7x)\tan(x^2 + 7x);\ g(x) = x^2 + 7x$
   $\Rightarrow g'(x) = 2x + 7$; therefore $\frac{dy}{dx} = f'(g(x))g'(x) = -(2x + 7)\sec(x^2 + 7x)\tan(x^2 + 7x)$

* Find the derivatives of the functions:

19. $p = \sqrt{3 - t}$
20. $q = \sqrt{2r - r^2}$
21. $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t$

22. $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right)$

23. $r = (\csc\theta + \cot\theta)^{-1}$ **24.** $r = -(\sec\theta + \tan\theta)^{-1}$

25. $y = x^2\sin^4 x + x\cos^{-2} x$ **26.** $y = \frac{1}{x}\sin^{-5} x - \frac{x}{3}\cos^3 x$

27. $y = \frac{1}{21}(3x-2)^7 + \left(4 - \frac{1}{2x^2}\right)^{-1}$

28. $y = (5-2x)^{-3} + \frac{1}{8}\left(\frac{2}{x}+1\right)^4$

29. $y = (4x+3)^4(x+1)^{-3}$ **30.** $y = (2x-5)^{-1}(x^2-5x)^6$

31. $h(x) = x\tan\left(2\sqrt{x}\right) + 7$ **32.** $k(x) = x^2\sec\left(\frac{1}{x}\right)$

33. $f(\theta) = \left(\frac{\sin\theta}{1+\cos\theta}\right)^2$ **34.** $g(t) = \left(\frac{1+\cos t}{\sin t}\right)^{-1}$

35. $r = \sin(\theta^2)\cos(2\theta)$ **36.** $r = \sec\sqrt{\theta}\tan\left(\frac{1}{\theta}\right)$

37. $q = \sin\left(\frac{t}{\sqrt{t+1}}\right)$ **38.** $q = \cot\left(\frac{\sin t}{t}\right)$

SOL :

19. $p = \sqrt{3-t} = (3-t)^{1/2} \Rightarrow \frac{dp}{dt} = \frac{1}{2}(3-t)^{-1/2}\cdot\frac{d}{dt}(3-t) = -\frac{1}{2}(3-t)^{-1/2} = \frac{-1}{2\sqrt{3-t}}$

20. $q = \sqrt{2r-r^2} = (2r-r^2)^{1/2} \Rightarrow \frac{dq}{dr} = \frac{1}{2}(2r-r^2)^{-1/2}\cdot\frac{d}{dr}(2r-r^2) = \frac{1}{2}(2r-r^2)^{-1/2}(2-2r) = \frac{1-r}{\sqrt{2r-r^2}}$

21. $s = \frac{4}{3\pi}\sin 3t + \frac{4}{5\pi}\cos 5t \Rightarrow \frac{ds}{dt} = \frac{4}{3\pi}\cos 3t\cdot\frac{d}{dt}(3t) + \frac{4}{5\pi}(-\sin 5t)\cdot\frac{d}{dt}(5t) = \frac{4}{\pi}\cos 3t - \frac{4}{\pi}\sin 5t$
$= \frac{4}{\pi}(\cos 3t - \sin 5t)$

22. $s = \sin\left(\frac{3\pi t}{2}\right) + \cos\left(\frac{3\pi t}{2}\right) \Rightarrow \frac{ds}{dt} = \cos\left(\frac{3\pi t}{2}\right)\cdot\frac{d}{dt}\left(\frac{3\pi t}{2}\right) - \sin\left(\frac{3\pi t}{2}\right)\cdot\frac{d}{dt}\left(\frac{3\pi t}{2}\right) = \frac{3\pi}{2}\cos\left(\frac{3\pi t}{2}\right) - \frac{3\pi}{2}\sin\left(\frac{3\pi t}{2}\right)$
$= \frac{3\pi}{2}\left(\cos\frac{3\pi t}{2} - \sin\frac{3\pi t}{2}\right)$

23. $r = (\csc\theta + \cot\theta)^{-1} \Rightarrow \frac{dr}{d\theta} = -(\csc\theta+\cot\theta)^{-2}\frac{d}{d\theta}(\csc\theta+\cot\theta) = \frac{\csc\theta\cot\theta+\csc^2\theta}{(\csc\theta+\cot\theta)^2} = \frac{\csc\theta(\cot\theta+\csc\theta)}{(\csc\theta+\cot\theta)^2}$
$= \frac{\csc\theta}{\csc\theta+\cot\theta}$

24. $r = -(\sec\theta+\tan\theta)^{-1} \Rightarrow \frac{dr}{d\theta} = (\sec\theta+\tan\theta)^{-2}\frac{d}{d\theta}(\sec\theta+\tan\theta) = \frac{\sec\theta\tan\theta+\sec^2\theta}{(\sec\theta+\tan\theta)^2} = \frac{\sec\theta(\tan\theta+\sec\theta)}{(\sec\theta+\tan\theta)^2}$
$= \frac{\sec\theta}{\sec\theta+\tan\theta}$

25. $y = x^2\sin^4 x + x\cos^{-2} x \Rightarrow \frac{dy}{dx} = x^2\frac{d}{dx}(\sin^4 x) + \sin^4 x\cdot\frac{d}{dx}(x^2) + x\frac{d}{dx}(\cos^{-2} x) + \cos^{-2} x\cdot\frac{d}{dx}(x)$
$= x^2\left(4\sin^3 x\frac{d}{dx}(\sin x)\right) + 2x\sin^4 x + x\left(-2\cos^{-3} x\cdot\frac{d}{dx}(\cos x)\right) + \cos^{-2} x$
$= x^2(4\sin^3 x\cos x) + 2x\sin^4 x + x((-2\cos^{-3} x)(-\sin x)) + \cos^{-2} x$
$= 4x^2\sin^3 x\cos x + 2x\sin^4 x + 2x\sin x\cos^{-3} x + \cos^{-2} x$

26. $y=\frac{1}{x}\sin^{-5}x-\frac{x}{3}\cos^3 x \Rightarrow y'=\frac{1}{x}\frac{d}{dx}(\sin^{-5}x)+\sin^{-5}x\cdot\frac{d}{dx}\left(\frac{1}{x}\right)-\frac{x}{3}\frac{d}{dx}(\cos^3 x)-\cos^3 x\cdot\frac{d}{dx}\left(\frac{x}{3}\right)$
$=\frac{1}{x}(-5\sin^{-6}x\cos x)+(\sin^{-5}x)\left(-\frac{1}{x^2}\right)-\frac{x}{3}\left((3\cos^2 x)(-\sin x)\right)-(\cos^3 x)\left(\frac{1}{3}\right)$
$=-\frac{5}{x}\sin^{-6}x\cos x-\frac{1}{x^2}\sin^{-5}x+x\cos^2 x\sin x-\frac{1}{3}\cos^3 x$

27. $y=\frac{1}{21}(3x-2)^7+\left(4-\frac{1}{2x^2}\right)^{-1} \Rightarrow \frac{dy}{dx}=\frac{7}{21}(3x-2)^6\cdot\frac{d}{dx}(3x-2)+(-1)\left(4-\frac{1}{2x^2}\right)^{-2}\cdot\frac{d}{dx}\left(4-\frac{1}{2x^2}\right)$
$=\frac{7}{21}(3x-2)^6\cdot 3+(-1)\left(4-\frac{1}{2x^2}\right)^{-2}\left(\frac{1}{x^3}\right)=(3x-2)^6-\frac{1}{x^3\left(4-\frac{1}{2x^2}\right)^2}$

28. $y=(5-2x)^{-3}+\frac{1}{8}\left(\frac{2}{x}+1\right)^4 \Rightarrow \frac{dy}{dx}=-3(5-2x)^{-4}(-2)+\frac{4}{8}\left(\frac{2}{x}+1\right)^3\left(-\frac{2}{x^2}\right)=6(5-2x)^{-4}-\left(\frac{1}{x^2}\right)\left(\frac{2}{x}+1\right)^3$
$=\frac{6}{(5-2x)^4}-\frac{\left(\frac{2}{x}+1\right)^3}{x^2}$

29. $y=(4x+3)^4(x+1)^{-3} \Rightarrow \frac{dy}{dx}=(4x+3)^4(-3)(x+1)^{-4}\cdot\frac{d}{dx}(x+1)+(x+1)^{-3}(4)(4x+3)^3\cdot\frac{d}{dx}(4x+3)$
$=(4x+3)^4(-3)(x+1)^{-4}(1)+(x+1)^{-3}(4)(4x+3)^3(4)=-3(4x+3)^4(x+1)^{-4}+16(4x+3)^3(x+1)^{-3}$
$=\frac{(4x+3)^3}{(x+1)^4}[-3(4x+3)+16(x+1)]=\frac{(4x+3)^3(4x+7)}{(x+1)^4}$

30. $y=(2x-5)^{-1}(x^2-5x)^6 \Rightarrow \frac{dy}{dx}=(2x-5)^{-1}(6)(x^2-5x)^5(2x-5)+(x^2-5x)^6(-1)(2x-5)^{-2}(2)$
$=6(x^2-5x)^5-\frac{2(x^2-5x)^6}{(2x-5)^2}$

31. $h(x)=x\tan\left(2\sqrt{x}\right)+7 \Rightarrow h'(x)=x\frac{d}{dx}\left(\tan\left(2x^{1/2}\right)\right)+\tan\left(2x^{1/2}\right)\cdot\frac{d}{dx}(x)+0$
$=x\sec^2\left(2x^{1/2}\right)\cdot\frac{d}{dx}\left(2x^{1/2}\right)+\tan\left(2x^{1/2}\right)=x\sec^2\left(2\sqrt{x}\right)\cdot\frac{1}{\sqrt{x}}+\tan\left(2\sqrt{x}\right)=\sqrt{x}\sec^2\left(2\sqrt{x}\right)+\tan\left(2\sqrt{x}\right)$

32. $k(x)=x^2\sec\left(\frac{1}{x}\right) \Rightarrow k'(x)=x^2\frac{d}{dx}\left(\sec\frac{1}{x}\right)+\sec\left(\frac{1}{x}\right)\cdot\frac{d}{dx}(x^2)=x^2\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\cdot\frac{d}{dx}\left(\frac{1}{x}\right)+2x\sec\left(\frac{1}{x}\right)$
$=x^2\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)\cdot\left(-\frac{1}{x^2}\right)+2x\sec\left(\frac{1}{x}\right)=2x\sec\left(\frac{1}{x}\right)-\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)$

33. $f(\theta)=\left(\frac{\sin\theta}{1+\cos\theta}\right)^2 \Rightarrow f'(\theta)=2\left(\frac{\sin\theta}{1+\cos\theta}\right)\cdot\frac{d}{d\theta}\left(\frac{\sin\theta}{1+\cos\theta}\right)=\frac{2\sin\theta}{1+\cos\theta}\cdot\frac{(1+\cos\theta)(\cos\theta)-(\sin\theta)(-\sin\theta)}{(1+\cos\theta)^2}$
$=\frac{(2\sin\theta)(\cos\theta+\cos^2\theta+\sin^2\theta)}{(1+\cos\theta)^3}=\frac{(2\sin\theta)(\cos\theta+1)}{(1+\cos\theta)^3}=\frac{2\sin\theta}{(1+\cos\theta)^2}$

34. $g(t)=\left(\frac{1+\cos t}{\sin t}\right)^{-1} \Rightarrow g'(t)=-\left(\frac{1+\cos t}{\sin t}\right)^{-2}\cdot\frac{d}{dt}\left(\frac{1+\cos t}{\sin t}\right)=-\frac{\sin^2 t}{(1+\cos t)^2}\cdot\frac{(\sin t)(-\sin t)-(1+\cos t)(\cos t)}{(\sin t)^2}$
$=\frac{-(-\sin^2 t-\cos t-\cos^2 t)}{(1+\cos t)^2}=\frac{1}{1+\cos t}$

35. $r=\sin(\theta^2)\cos(2\theta) \Rightarrow \frac{dr}{d\theta}=\sin(\theta^2)(-\sin 2\theta)\frac{d}{d\theta}(2\theta)+\cos(2\theta)(\cos(\theta^2))\cdot\frac{d}{d\theta}(\theta^2)$
$=\sin(\theta^2)(-\sin 2\theta)(2)+(\cos 2\theta)(\cos(\theta^2))(2\theta)=-2\sin(\theta^2)\sin(2\theta)+2\theta\cos(2\theta)\cos(\theta^2)$

36. $r=\left(\sec\sqrt{\theta}\right)\tan\left(\frac{1}{\theta}\right) \Rightarrow \frac{dr}{d\theta}=\left(\sec\sqrt{\theta}\right)\left(\sec^2\frac{1}{\theta}\right)\left(-\frac{1}{\theta^2}\right)+\tan\left(\frac{1}{\theta}\right)\left(\sec\sqrt{\theta}\tan\sqrt{\theta}\right)\left(\frac{1}{2\sqrt{\theta}}\right)$
$=-\frac{1}{\theta^2}\sec\sqrt{\theta}\sec^2\left(\frac{1}{\theta}\right)+\frac{1}{2\sqrt{\theta}}\tan\left(\frac{1}{\theta}\right)\sec\sqrt{\theta}\tan\sqrt{\theta}=\left(\sec\sqrt{\theta}\right)\left[\frac{\tan\sqrt{\theta}\tan\left(\frac{1}{\theta}\right)}{2\sqrt{\theta}}-\frac{\sec^2\left(\frac{1}{\theta}\right)}{\theta^2}\right]$

37. $q=\sin\left(\frac{t}{\sqrt{t+1}}\right) \Rightarrow \frac{dq}{dt}=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{d}{dt}\left(\frac{t}{\sqrt{t+1}}\right)=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{\sqrt{t+1}(1)-t\cdot\frac{d}{dt}\left(\sqrt{t+1}\right)}{\left(\sqrt{t+1}\right)^2}$
$=\cos\left(\frac{t}{\sqrt{t+1}}\right)\cdot\frac{\sqrt{t+1}-\frac{t}{2\sqrt{t+1}}}{t+1}=\cos\left(\frac{t}{\sqrt{t+1}}\right)\left(\frac{2(t+1)-t}{2(t+1)^{3/2}}\right)=\left(\frac{t+2}{2(t+1)^{3/2}}\right)\cos\left(\frac{t}{\sqrt{t+1}}\right)$

38. $q=\cot\left(\frac{\sin t}{t}\right) \Rightarrow \frac{dq}{dt}=-\csc^2\left(\frac{\sin t}{t}\right)\cdot\frac{d}{dt}\left(\frac{\sin t}{t}\right)=\left(-\csc^2\left(\frac{\sin t}{t}\right)\right)\left(\frac{t\cos t-\sin t}{t^2}\right)$

*find dy/dt

39. $y = \sin^2(\pi t - 2)$ 40. $y = \sec^2 \pi t$

41. $y = (1 + \cos 2t)^{-4}$ 42. $y = (1 + \cot(t/2))^{-2}$

43. $y = \sin(\cos(2t - 5))$ 44. $y = \cos\left(5 \sin\left(\frac{t}{3}\right)\right)$

45. $y = \left(1 + \tan^4\left(\frac{t}{12}\right)\right)^3$ 46. $y = \frac{1}{6}\left(1 + \cos^2(7t)\right)^3$

47. $y = \sqrt{1 + \cos(t^2)}$ 48. $y = 4\sin\left(\sqrt{1 + \sqrt{t}}\right)$

SOL :

39. $y = \sin^2(\pi t - 2) \Rightarrow \frac{dy}{dt} = 2\sin(\pi t - 2) \cdot \frac{d}{dt}\sin(\pi t - 2) = 2\sin(\pi t - 2) \cdot \cos(\pi t - 2) \cdot \frac{d}{dt}(\pi t - 2)$
$= 2\pi \sin(\pi t - 2)\cos(\pi t - 2)$

40. $y = \sec^2 \pi t \Rightarrow \frac{dy}{dt} = (2\sec \pi t) \cdot \frac{d}{dt}(\sec \pi t) = (2\sec \pi t)(\sec \pi t \tan \pi t) \cdot \frac{d}{dt}(\pi t) = 2\pi \sec^2 \pi t \tan \pi t$

41. $y = (1 + \cos 2t)^{-4} \Rightarrow \frac{dy}{dt} = -4(1 + \cos 2t)^{-5} \cdot \frac{d}{dt}(1 + \cos 2t) = -4(1 + \cos 2t)^{-5}(-\sin 2t) \cdot \frac{d}{dt}(2t) = \frac{8 \sin 2t}{(1 + \cos 2t)^5}$

42. $y = \left(1 + \cot\left(\frac{t}{2}\right)\right)^{-2} \Rightarrow \frac{dy}{dt} = -2\left(1 + \cot\left(\frac{t}{2}\right)\right)^{-3} \cdot \frac{d}{dt}\left(1 + \cot\left(\frac{t}{2}\right)\right) = -2\left(1 + \cot\left(\frac{t}{2}\right)\right)^{-3} \cdot \left(-\csc^2\left(\frac{t}{2}\right)\right) \cdot \frac{d}{dt}\left(\frac{t}{2}\right)$
$= \frac{\csc^2\left(\frac{t}{2}\right)}{\left(1 + \cot\left(\frac{t}{2}\right)\right)^3}$

43. $y = \sin(\cos(2t - 5)) \Rightarrow \frac{dy}{dt} = \cos(\cos(2t - 5)) \cdot \frac{d}{dt}\cos(2t - 5) = \cos(\cos(2t - 5)) \cdot (-\sin(2t - 5)) \cdot \frac{d}{dt}(2t - 5)$
$= -2\cos(\cos(2t - 5))(\sin(2t - 5))$

44. $y = \cos\left(5\sin\left(\frac{t}{3}\right)\right) \Rightarrow \frac{dy}{dt} = -\sin\left(5\sin\left(\frac{t}{3}\right)\right) \cdot \frac{d}{dt}\left(5\sin\left(\frac{t}{3}\right)\right) = -\sin\left(5\sin\left(\frac{t}{3}\right)\right)\left(5\cos\left(\frac{t}{3}\right)\right) \cdot \frac{d}{dt}\left(\frac{t}{3}\right)$
$= -\frac{5}{3}\sin\left(5\sin\left(\frac{t}{3}\right)\right)\left(\cos\left(\frac{t}{3}\right)\right)$

45. $y = \left[1 + \tan^4\left(\frac{t}{12}\right)\right]^3 \Rightarrow \frac{dy}{dt} = 3\left[1 + \tan^4\left(\frac{t}{12}\right)\right]^2 \cdot \frac{d}{dt}\left[1 + \tan^4\left(\frac{t}{12}\right)\right] = 3\left[1 + \tan^4\left(\frac{t}{12}\right)\right]^2\left[4\tan^3\left(\frac{t}{12}\right) \cdot \frac{d}{dt}\tan\left(\frac{t}{12}\right)\right]$
$= 12\left[1 + \tan^4\left(\frac{t}{12}\right)\right]^2\left[\tan^3\left(\frac{t}{12}\right)\sec^2\left(\frac{t}{12}\right) \cdot \frac{1}{12}\right] = \left[1 + \tan^4\left(\frac{t}{12}\right)\right]^2\left[\tan^3\left(\frac{t}{12}\right)\sec^2\left(\frac{t}{12}\right)\right]$

46. $y = \frac{1}{6}\left[1 + \cos^2(7t)\right]^3 \Rightarrow \frac{dy}{dt} = \frac{3}{6}\left[1 + \cos^2(7t)\right]^2 \cdot 2\cos(7t)(-\sin(7t))(7) = -7\left[1 + \cos^2(7t)\right]^2(\cos(7t)\sin(7t))$

47. $y = (1 + \cos(t^2))^{1/2} \Rightarrow \frac{dy}{dt} = \frac{1}{2}(1 + \cos(t^2))^{-1/2} \cdot \frac{d}{dt}(1 + \cos(t^2)) = \frac{1}{2}(1 + \cos(t^2))^{-1/2}\left(-\sin(t^2) \cdot \frac{d}{dt}(t^2)\right)$
$= -\frac{1}{2}(1 + \cos(t^2))^{-1/2}(\sin(t^2)) \cdot 2t = -\frac{t\sin(t^2)}{\sqrt{1 + \cos(t^2)}}$

48. $y = 4\sin\left(\sqrt{1 + \sqrt{t}}\right) \Rightarrow \frac{dy}{dt} = 4\cos\left(\sqrt{1 + \sqrt{t}}\right) \cdot \frac{d}{dt}\left(\sqrt{1 + \sqrt{t}}\right) = 4\cos\left(\sqrt{1 + \sqrt{t}}\right) \cdot \frac{1}{2\sqrt{1 + \sqrt{t}}} \cdot \frac{d}{dt}\left(1 + \sqrt{t}\right)$
$= \frac{2\cos\left(\sqrt{1 + \sqrt{t}}\right)}{\sqrt{1 + \sqrt{t}} \cdot 2\sqrt{t}} = \frac{\cos\left(\sqrt{1 + \sqrt{t}}\right)}{\sqrt{t + \sqrt{t}}}$

*Find y'' :

49. $y=\left(1+\frac{1}{x}\right)^3$ **50.** $y=\left(1-\sqrt{x}\right)^{-1}$

51. $y=\frac{1}{9}\cot(3x-1)$ **52.** $y=9\tan\left(\frac{x}{3}\right)$

SOL :

49. $y=\left(1+\frac{1}{x}\right)^3 \Rightarrow y'=3\left(1+\frac{1}{x}\right)^2\left(-\frac{1}{x^2}\right)=-\frac{3}{x^2}\left(1+\frac{1}{x}\right)^2 \Rightarrow y''=\left(-\frac{3}{x^2}\right)\cdot\frac{d}{dx}\left(1+\frac{1}{x}\right)^2-\left(1+\frac{1}{x}\right)^2\cdot\frac{d}{dx}\left(\frac{3}{x^2}\right)$
$=\left(-\frac{3}{x^2}\right)\left(2\left(1+\frac{1}{x}\right)\left(-\frac{1}{x^2}\right)\right)+\left(\frac{6}{x^3}\right)\left(1+\frac{1}{x}\right)^2=\frac{6}{x^4}\left(1+\frac{1}{x}\right)+\frac{6}{x^3}\left(1+\frac{1}{x}\right)^2=\frac{6}{x^3}\left(1+\frac{1}{x}\right)\left(\frac{1}{x}+1+\frac{1}{x}\right)$
$=\frac{6}{x^3}\left(1+\frac{1}{x}\right)\left(1+\frac{2}{x}\right)$

50. $y=\left(1-\sqrt{x}\right)^{-1} \Rightarrow y'=-\left(1-\sqrt{x}\right)^{-2}\left(-\frac{1}{2}x^{-1/2}\right)=\frac{1}{2}\left(1-\sqrt{x}\right)^{-2}x^{-1/2}$
$\Rightarrow y''=\frac{1}{2}\left[\left(1-\sqrt{x}\right)^{-2}\left(-\frac{1}{2}x^{-3/2}\right)+x^{-1/2}(-2)\left(1-\sqrt{x}\right)^{-3}\left(-\frac{1}{2}x^{-1/2}\right)\right]$
$=\frac{1}{2}\left[\frac{-1}{2}x^{-3/2}\left(1-\sqrt{x}\right)^{-2}+x^{-1}\left(1-\sqrt{x}\right)^{-3}\right]=\frac{1}{2}x^{-1}\left(1-\sqrt{x}\right)^{-3}\left[-\frac{1}{2}x^{-1/2}\left(1-\sqrt{x}\right)+1\right]$
$=\frac{1}{2x}\left(1-\sqrt{x}\right)^{-3}\left(-\frac{1}{2\sqrt{x}}+\frac{1}{2}+1\right)=\frac{1}{2x}\left(1-\sqrt{x}\right)^{-3}\left(\frac{3}{2}-\frac{1}{2\sqrt{x}}\right)$

51. $y=\frac{1}{9}\cot(3x-1) \Rightarrow y'=-\frac{1}{9}\csc^2(3x-1)(3)=-\frac{1}{3}\csc^2(3x-1) \Rightarrow y''=\left(-\frac{2}{3}\right)\left(\csc(3x-1)\cdot\frac{d}{dx}\csc(3x-1)\right)$
$=-\frac{2}{3}\csc(3x-1)(-\csc(3x-1)\cot(3x-1)\cdot\frac{d}{dx}(3x-1))=2\csc^2(3x-1)\cot(3x-1)$

52. $y=9\tan\left(\frac{x}{3}\right) \Rightarrow y'=9\left(\sec^2\left(\frac{x}{3}\right)\right)\left(\frac{1}{3}\right)=3\sec^2\left(\frac{x}{3}\right) \Rightarrow y''=3\cdot 2\sec\left(\frac{x}{3}\right)\left(\sec\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)\right)\left(\frac{1}{3}\right)=2\sec^2\left(\frac{x}{3}\right)\tan\left(\frac{x}{3}\right)$

*find the value of $(f\circ g)'$ at the given value of x :

53. $f(u)=u^5+1, \quad u=g(x)=\sqrt{x}, \quad x=1$

54. $f(u)=1-\frac{1}{u}, \quad u=g(x)=\frac{1}{1-x}, \quad x=-1$

55. $f(u)=\cot\frac{\pi u}{10}, \quad u=g(x)=5\sqrt{x}, \quad x=1$

56. $f(u)=u+\frac{1}{\cos^2 u}, \quad u=g(x)=\pi x, \quad x=1/4$

57. $f(u)=\frac{2u}{u^2+1}, \quad u=g(x)=10x^2+x+1, \quad x=0$

58. $f(u)=\left(\frac{u-1}{u+1}\right)^2, \quad u=g(x)=\frac{1}{x^2}-1, \quad x=-1$

SOL :

53. $g(x)=\sqrt{x} \Rightarrow g'(x)=\frac{1}{2\sqrt{x}} \Rightarrow g(1)=1$ and $g'(1)=\frac{1}{2}$; $f(u)=u^5+1 \Rightarrow f'(u)=5u^4 \Rightarrow f'(g(1))=f'(1)=5$;
therefore, $(f\circ g)'(1)=f'(g(1))\cdot g'(1)=5\cdot\frac{1}{2}=\frac{5}{2}$

54. $g(x) = (1-x)^{-1} \Rightarrow g'(x) = -(1-x)^{-2}(-1) = \frac{1}{(1-x)^2} \Rightarrow g(-1) = \frac{1}{2}$ and $g'(-1) = \frac{1}{4}$; $f(u) = 1 - \frac{1}{u}$
$\Rightarrow f'(u) = \frac{1}{u^2} \Rightarrow f'(g(-1)) = f'\left(\frac{1}{2}\right) = 4$; therefore, $(f \circ g)'(-1) = f'(g(-1))g'(-1) = 4 \cdot \frac{1}{4} = 1$

55. $g(x) = 5\sqrt{x} \Rightarrow g'(x) = \frac{5}{2\sqrt{x}} \Rightarrow g(1) = 5$ and $g'(1) = \frac{5}{2}$; $f(u) = \cot\left(\frac{\pi u}{10}\right) \Rightarrow f'(u) = -\csc^2\left(\frac{\pi u}{10}\right)\left(\frac{\pi}{10}\right)$
$= \frac{-\pi}{10}\csc^2\left(\frac{\pi u}{10}\right) \Rightarrow f'(g(1)) = f'(5) = -\frac{\pi}{10}\csc^2\left(\frac{\pi}{2}\right) = -\frac{\pi}{10}$; therefore, $(f \circ g)'(1) = f'(g(1))g'(1) = -\frac{\pi}{10} \cdot \frac{5}{2}$
$= -\frac{\pi}{4}$

56. $g(x) = \pi x \Rightarrow g'(x) = \pi \Rightarrow g\left(\frac{1}{4}\right) = \frac{\pi}{4}$ and $g'\left(\frac{1}{4}\right) = \pi$; $f(u) = u + \sec^2 u \Rightarrow f'(u) = 1 + 2\sec u \cdot \sec u \tan u$
$= 1 + 2\sec^2 u \tan u \Rightarrow f'\left(g\left(\frac{1}{4}\right)\right) = f'\left(\frac{\pi}{4}\right) = 1 + 2\sec^2\frac{\pi}{4}\tan\frac{\pi}{4} = 5$; therefore, $(f \circ g)'\left(\frac{1}{4}\right) = f'\left(g\left(\frac{1}{4}\right)\right)g'\left(\frac{1}{4}\right) = 5\pi$

57. $g(x) = 10x^2 + x + 1 \Rightarrow g'(x) = 20x + 1 \Rightarrow g(0) = 1$ and $g'(0) = 1$; $f(u) = \frac{2u}{u^2+1} \Rightarrow f'(u) = \frac{(u^2+1)(2) - (2u)(2u)}{(u^2+1)^2}$
$= \frac{-2u^2+2}{(u^2+1)^2} \Rightarrow f'(g(0)) = f'(1) = 0$; therefore, $(f \circ g)'(0) = f'(g(0))g'(0) = 0 \cdot 1 = 0$

58. $g(x) = \frac{1}{x^2} - 1 \Rightarrow g'(x) = -\frac{2}{x^3} \Rightarrow g(-1) = 0$ and $g'(-1) = 2$; $f(u) = \left(\frac{u-1}{u+1}\right)^2 \Rightarrow f'(u) = 2\left(\frac{u-1}{u+1}\right)\frac{d}{du}\left(\frac{u-1}{u+1}\right)$
$= 2\left(\frac{u-1}{u+1}\right) \cdot \frac{(u+1)(1)-(u-1)(1)}{(u+1)^2} = \frac{2(u-1)(2)}{(u+1)^3} = \frac{4(u-1)}{(u+1)^3} \Rightarrow f'(g(-1)) = f'(0) = -4$; therefore,
$(f \circ g)'(-1) = f'(g(-1))g'(-1) = (-4)(2) = -8$

EX :

63. Find dy/dx if $y = x$ by using the Chain Rule with y as a composite of

a. $y = (u/5) + 7$ and $u = 5x - 35$

b. $y = 1 + (1/u)$ and $u = 1/(x-1)$.

64. Find dy/dx if $y = x^{3/2}$ by using the Chain Rule with y as a composite of

a. $y = u^3$ and $u = \sqrt{x}$

b. $y = \sqrt{u}$ and $u = x^3$.

63. With $y = x$, we should get $\frac{dy}{dx} = 1$ for both (a) and (b):

(a) $y = \frac{u}{5} + 7 \Rightarrow \frac{dy}{du} = \frac{1}{5}$; $u = 5x - 35 \Rightarrow \frac{du}{dx} = 5$; therefore, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{5} \cdot 5 = 1$, as expected

(b) $y = 1 + \frac{1}{u} \Rightarrow \frac{dy}{du} = -\frac{1}{u^2}$; $u = (x-1)^{-1} \Rightarrow \frac{du}{dx} = -(x-1)^{-2}(1) = \frac{-1}{(x-1)^2}$; therefore $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$= \frac{-1}{u^2} \cdot \frac{-1}{(x-1)^2} = \frac{-1}{((x-1)^{-1})^2} \cdot \frac{-1}{(x-1)^2} = (x-1)^2 \cdot \frac{1}{(x-1)^2} = 1$, again as expected

64. With $y = x^{3/2}$, we should get $\frac{dy}{dx} = \frac{3}{2}x^{1/2}$ for both (a) and (b):

(a) $y = u^3 \Rightarrow \frac{dy}{du} = 3u^2$; $u = \sqrt{x} \Rightarrow \frac{du}{dx} = \frac{1}{2\sqrt{x}}$; therefore, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2 \cdot \frac{1}{2\sqrt{x}} = 3\left(\sqrt{x}\right)^2 \cdot \frac{1}{2\sqrt{x}} = \frac{3}{2}\sqrt{x}$, as expected.

(b) $y = \sqrt{u} \Rightarrow \frac{dy}{du} = \frac{1}{2\sqrt{u}}$; $u = x^3 \Rightarrow \frac{du}{dx} = 3x^2$; therefore, $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} \cdot 3x^2 = \frac{1}{2\sqrt{x^3}} \cdot 3x^2 = \frac{3}{2}x^{1/2}$, again as expected.