EXAMPLES

Evaluate the following integrals by using tabular integration :

1)

$$\int p^4 e^{-p}\, dp$$

SOL :

$$\begin{array}{lcl} & & e^{-p} \\ p^4 & \xrightarrow{(+)} & -e^{-p} \\ 4p^3 & \xrightarrow{(-)} & e^{-p} \\ 12p^2 & \xrightarrow{(+)} & -e^{-p} \\ 24p & \xrightarrow{(-)} & e^{-p} \\ 24 & \xrightarrow{(+)} & -e^{-p} \\ 0 & & \end{array}$$

$$\int p^4 e^{-p}\, dp = -p^4 e^{-p} - 4p^3 e^{-p} - 12p^2 e^{-p} - 24pe^{-p} - 24e^{-p} + C$$
$$= (-p^4 - 4p^3 - 12p^2 - 24p - 24)\, e^{-p} + C$$

2)

$$\int (x^2 - 5x)e^x\, dx$$

SOL :

$$\begin{array}{lcl} & & e^{x} \\ x^2 - 5x & \xrightarrow{(+)} & e^{x} \\ 2x - 5 & \xrightarrow{(-)} & e^{x} \\ 2 & \xrightarrow{(+)} & e^{x} \\ 0 & & \end{array}$$

$$\int (x^2 - 5x)\, e^x\, dx = (x^2 - 5x)\, e^x - (2x - 5)e^x + 2e^x + C = x^2 e^x - 7xe^x + 7e^x + C$$
$$= (x^2 - 7x + 7)\, e^x + C$$

3)

$$\int (r^2 + r + 1)e^r\, dr$$

SOL :

$$\begin{array}{lcl} & & e^{r} \\ r^2 + r + 1 & \xrightarrow{(+)} & e^{r} \\ 2r + 1 & \xrightarrow{(-)} & e^{r} \\ 2 & \xrightarrow{(+)} & e^{r} \\ 0 & & \end{array}$$

$$\int (r^2 + r + 1)\, e^r\, dr = (r^2 + r + 1)\, e^r - (2r + 1)\, e^r + 2e^r + C$$
$$= [(r^2 + r + 1) - (2r + 1) + 2]\, e^r + C = (r^2 - r + 2)\, e^r + C$$

4) $\int x^5 e^x \, dx$

SOL :

$$
\begin{array}{lcl}
 & & e^x \\
x^5 & \xrightarrow{(+)} & e^x \\
5x^4 & \xrightarrow{(-)} & e^x \\
20x^3 & \xrightarrow{(+)} & e^x \\
60x^2 & \xrightarrow{(-)} & e^x \\
120x & \xrightarrow{(+)} & e^x \\
120 & \xrightarrow{(-)} & e^x \\
0 & &
\end{array}
$$

$$\int x^5 e^x \, dx = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120 e^x + C$$
$$= (x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120)\, e^x + C$$

5)

$$\int t^2 e^{4t} \, dt$$

SOL :

$$
\begin{array}{lcl}
 & & e^{4t} \\
t^2 & \xrightarrow{(+)} & \frac{1}{4} e^{4t} \\
2t & \xrightarrow{(-)} & \frac{1}{16} e^{4t} \\
2 & \xrightarrow{(+)} & \frac{1}{64} e^{4t} \\
0 & &
\end{array}
$$

$$\int t^2 e^{4t} \, dt = \frac{t^2}{4} e^{4t} - \frac{2t}{16} e^{4t} + \frac{2}{64} e^{4t} + C = \frac{t^2}{4} e^{4t} - \frac{t}{8} e^{4t} + \frac{1}{32} e^{4t} + C$$
$$\left(\frac{t^2}{4} - \frac{t}{8} + \frac{1}{32}\right) e^{4t} + C$$

6) $\int_0^{\pi/2} \theta^2 \sin 2\theta \, d\theta$

SOL :

$$
\begin{array}{lcl}
 & & \sin 2\theta \\
\theta^2 & \xrightarrow{(+)} & -\frac{1}{2} \cos 2\theta \\
2\theta & \xrightarrow{(-)} & -\frac{1}{4} \sin 2\theta \\
2 & \xrightarrow{(+)} & \frac{1}{8} \cos 2\theta \\
0 & &
\end{array}
$$

$$\int_0^{\pi/2} \theta^2 \sin 2\theta \, d\theta = \left[-\frac{\theta^2}{2} \cos 2\theta + \frac{\theta}{2} \sin 2\theta + \frac{1}{4} \cos 2\theta \right]_0^{\pi/2}$$
$$= \left[-\frac{\pi^2}{8} \cdot (-1) + \frac{\pi}{4} \cdot 0 + \frac{1}{4} \cdot (-1) \right] - \left[0 + 0 + \frac{1}{4} \cdot 1 \right] = \frac{\pi^2}{8} - \frac{1}{2} = \frac{\pi^2 - 4}{8}$$

7)

$$\int_0^{\pi/2} x^3 \cos 2x \, dx$$

SOL :

$$\begin{array}{lcl}
 & & \cos 2x \\
x^3 & \xrightarrow{(+)} & \frac{1}{2}\sin 2x \\
3x^2 & \xrightarrow{(-)} & -\frac{1}{4}\cos 2x \\
6x & \xrightarrow{(+)} & -\frac{1}{8}\sin 2x \\
6 & \xrightarrow{(-)} & \frac{1}{16}\cos 2x \\
0 & &
\end{array}$$

$$\int_0^{\pi/2} x^3 \cos 2x \, dx = \left[\frac{x^3}{2}\sin 2x + \frac{3x^2}{4}\cos 2x - \frac{3x}{4}\sin 2x - \frac{3}{8}\cos 2x\right]_0^{\pi/2}$$

$$= \left[\frac{\pi^3}{16}\cdot 0 + \frac{3\pi^2}{16}\cdot(-1) - \frac{3\pi}{8}\cdot 0 - \frac{3}{8}\cdot(-1)\right] - \left[0 + 0 - 0 - \frac{3}{8}\cdot 1\right] = -\frac{3\pi^2}{16} + \frac{3}{4} = \frac{3(4-\pi^2)}{16}$$

8)

$$\int z(\ln z)^2 \, dz$$

SOL :

$$\int z(\ln z)^2 \, dz; \begin{bmatrix} u = \ln z \\ du = \frac{1}{z}\,dz \\ dz = e^u \, du \end{bmatrix} \to \int e^u \cdot u^2 \cdot e^u \, du = \int e^{2u} \cdot u^2 \, du;$$

$$\begin{array}{lcl}
 & & e^{2u} \\
u^2 & \xrightarrow{(+)} & \frac{1}{2}e^{2u} \\
2u & \xrightarrow{(-)} & \frac{1}{4}e^{2u} \\
2 & \xrightarrow{(+)} & \frac{1}{8}e^{2u} \\
0 & &
\end{array}$$

$$\int u^2 e^{2u} \, du = \frac{u^2}{2}e^{2u} - \frac{u}{2}e^{2u} + \frac{1}{4}e^{2u} + C = \frac{e^{2u}}{4}\left[2u^2 - 2u + 1\right] + C$$

$$= \frac{z^2}{4}\left[2(\ln z)^2 - 2\ln z + 1\right] + C$$