L Hopital's Rule

Suppose that $f(a)=g(a)=0$ **that** $f'(a)$ **and** $g'(a)$ **exist , and that** $g'(a)\neq 0$

Then $\lim\limits_{x\Rightarrow a}\dfrac{f(x)}{g(x)}=\dfrac{f'(a)}{f'(a)}$ **[H.W. (prove it)**

and $\lim\limits_{x\Rightarrow a}\dfrac{f(x)}{g(x)}=\lim\dfrac{f'(x)}{f'(x)}$

Find the following limits by using L Hopital'Rule :

(a) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-1-x/2}{x^2}$ $\qquad\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{(1/2)(1+x)^{-1/2}-1/2}{2x}\qquad \text{Still } \frac{0}{0}\text{; differentiate agai}$$

$$=\lim_{x\to 0}\frac{-(1/4)(1+x)^{-3/2}}{2}=-\frac{1}{8}\qquad \text{Not } \frac{0}{0}\text{; limit is found.}$$

(b) $\lim\limits_{x\to 0}\dfrac{x-\sin x}{x^3}$ $\qquad\dfrac{0}{0}$

$$=\lim_{x\to 0}\frac{1-\cos x}{3x^2}\qquad \text{Still } \frac{0}{0}$$

$$=\lim_{x\to 0}\frac{\sin x}{6x}\qquad \text{Still } \frac{0}{0}$$

$$=\lim_{x\to 0}\frac{\cos x}{6}=\frac{1}{6}\qquad \text{Not } \frac{0}{0}\text{; limit is found.}$$

Find The Following Limits (By using L Hopital's Rule)

1- $\lim\limits_{x\Rightarrow\infty}\dfrac{2x-2}{3x-2}=\dfrac{\infty-1}{\infty-2}=\dfrac{\infty}{\infty}$ *meaning less* $\lim\limits_{x\Rightarrow\infty}\dfrac{2}{3}=\dfrac{2}{3}$

2- $\lim\limits_{x\Rightarrow 0}\dfrac{2x^2+3x}{5x^2-2x}=\dfrac{2(0)^2+3(0)}{5(0)^2-2(0)}=\dfrac{0}{0}$ *meaning less* $\Rightarrow\lim\limits_{x\Rightarrow 0}\dfrac{4x+3}{10x-2}=\dfrac{4(0)+3}{10(0)-2}=\dfrac{-3}{2}$

3- $\lim\limits_{x\Rightarrow 0}\dfrac{\sqrt{1+x}-1}{x}=\dfrac{\sqrt{1+0}-1}{0}=\dfrac{0}{0}$ *meaning less* $\Rightarrow\lim\limits_{x\Rightarrow 0}\dfrac{1/2(1+x)^{-1/2}}{10x-2}=\dfrac{1}{2}(1+0)^{-1/2}=\dfrac{1}{2}$

4- $\lim\limits_{x\Rightarrow 1}\dfrac{x^3-3x+2}{x^3-x^2-x+1}=\dfrac{(1)^3-3(1)+2}{(1)^3-(1)^2-(1)+1}=\dfrac{-2+2}{-1+1}=\dfrac{0}{0}$ *meaning less*

$$\lim_{x\Rightarrow 1}\frac{3x^2-3}{3x^2-2x-1}=\frac{3-3}{3-2-1}=\frac{0}{0}\Rightarrow\lim_{x\Rightarrow 1}\frac{6x}{6x-2}=\frac{6}{4}=\frac{3}{2}$$

5- $\lim\limits_{x \Rightarrow \pi/2} [\sec(x) - \tan(x)] = \lim\limits_{x \Rightarrow \pi/2} \left(\frac{1}{\cos(x)} - \frac{\sin(x)}{\cos(x)} \right) \Rightarrow \lim\limits_{x \Rightarrow \pi/2} \frac{1 - \sin(x)}{\cos(x)} = \frac{1 - \sin(\pi/2)}{\cos(\pi/2)} = \frac{1-1}{0} = \frac{0}{0}$

$\Rightarrow \lim\limits_{x \Rightarrow \pi/2} \frac{-\cos(x)}{-\sin(x)} = \frac{\cos(\pi/2)}{\sin(\pi/2)} = \frac{0}{1} = 0$

6- $\lim\limits_{x \Rightarrow 0} \frac{\sin(x)}{x} = \frac{\sin(0)}{0} = \frac{0}{0}$ *meaning less* $\Rightarrow \lim\limits_{x \Rightarrow 0} \frac{\cos(x)}{1} = \frac{\cos(0)}{1} = \frac{1}{1} = 1$

7- $\lim\limits_{x \Rightarrow 0} \frac{\ln(x+1) - 2x}{x^2} = \frac{\ln(0+1) - 2(0)}{(0)^2} = \frac{\ln(1) - 0}{0} = \frac{0}{0}$ *meaning less* $\lim\limits_{x \Rightarrow 0} \frac{\frac{1}{(x+1)} - 2}{2x} = \frac{\frac{1}{0+1} - 2}{2(0)} = \frac{-1}{0} = \infty$

8- $\lim\limits_{x \Rightarrow 0} \frac{e^{2x} - 2x - 1}{1 - \cos(x)} = \frac{e^0 - 0 - 1}{1 - \cos(0)} = \frac{1-1}{1-1} = \frac{0}{0}$ *meaning less*

$\Rightarrow \lim\limits_{x \Rightarrow 0} \frac{2e^{2x} - 2}{\sin(x)} = \frac{2e^0 - 2}{\sin(0)} = \frac{2-2}{0} = \frac{0}{0}$ *meaning less* $\Rightarrow \lim\limits_{x \Rightarrow 0} \frac{4e^{2x}}{\cos(x)} = \frac{4e^0}{\cos(0)} = \frac{4}{1} = 4$

9- $\lim\limits_{x \Rightarrow 0} \frac{\sin(2x)}{\sin(3x)} = \frac{\sin(0)}{\sin(0)} = \frac{0}{0}$ *meaning less* $\lim\limits_{x \Rightarrow 0} \frac{2\cos(2x)}{3\cos(3x)} = \frac{2\cos(0)}{3\cos(0)} = \frac{2}{3}$

H.W : Find the following limits :

$\lim\limits_{x \Rightarrow 0} \frac{x-2}{x^2 - 4}$ $\quad \lim\limits_{t \Rightarrow \infty} \frac{6t+5}{3t-8}$ $\quad \lim\limits_{x \Rightarrow \pi/2} \frac{2x - \pi}{\cos(x)}$ $\quad \lim\limits_{t \Rightarrow \infty} \frac{t^2 - 2t}{3t^2 - 2}$

$\lim\limits_{x \Rightarrow \pi/3} \frac{\cos(x) - 1/2}{x - \pi/3}$ $\quad \lim\limits_{\theta \Rightarrow \pi} \frac{\sin(\theta)}{\pi - \theta}$ $\quad \lim\limits_{x \Rightarrow 1} \frac{x^3 - 1}{4x^3 - x - 3}$ $\quad \lim\limits_{t \Rightarrow 0} \frac{\cos(t) - 1}{t^2}$

$1 - \sinh(0) = \frac{e^0 - e^{-0}}{2} = \frac{1-1}{2} = 0$

$2 - \sin(30) = \sin(\frac{\pi}{6}) = \frac{1}{2}$

$3 - \sin(90) = \sin(\frac{\pi}{2}) = 1$

$4 - \; 60 = \tan^{-1}(\theta) \Rightarrow \theta = \tan(60) \Rightarrow \theta = \frac{\sin(60)}{\cos(60)} = \frac{\sqrt{3}/2}{1/2} = \frac{0.866025}{0.5} = 1.73205$

1 Show That

$\sinh(x)+\cosh(x)=e^{x}$	$\tan(a+b)=\dfrac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$	$\sin(a-\dfrac{\pi}{2})=-\cos(x)$

2 Derivative

$$y^4+\tan(xy)+\sin(x)=2$$

$$y=\sinh^2\left[\sec^{-1}(2x)\right].\cos^{-1}(\frac{1}{x})$$

$$y=\sin^4[\ln(x)]\tan^{-1}(x^2+1)$$

$$y=2^{\sin^{-1}(3x+1)}.e^{x^2+1}$$

$$y=\tan^{-1}[\sin(x)].\tanh^3(e^{2x})$$

$$y=\frac{\ln(x^2+1),\sec(x+2).e^{x^2+1}}{2^x(x+1)^2\tan^{-1}(e^x)}$$

$$y=\tan\left[\sec^3\left(x^2+\sin(2x)\right)\right]^{-2}$$

$$y=\left[\ln(x^2+\sin^{-1}(2x)\right]^3$$

$$y=e^{2\sin(3x)}.\csc^{-1}(x^2+2x+1)$$

$$y=\tanh\left|x^2+\sin(2x)\right|$$

$$y=\sinh^2\left|\sin^3(x^2+2x+3)\right|$$

$$y=\sec h^{-2}\left|\sec h(x^2+1)\right|\sin^{-1}[x^2+\csc h(x)]$$

$$y=\frac{1}{x^2-4}\quad,\quad y=\sqrt{x-1}\quad,\quad y=\sin(x)$$