Methods Of Integration

Integral Formulas (Standard Forms)

1	$\int u^n du = \frac{u^{n+1}}{n+1} + c \quad n \neq -1$	$\int \frac{du}{u} = \ln(u) + c$
2	$\int e^u du = e^u + c$	$\int a^u du = \frac{a^u}{\ln(a)} + c$ **a is constant**
3	$\int \sin(u) du = -\cos(u) + c$	$\int \sinh(u) du = \cosh(u) + c$
4	$\int \cos(u) du = \sin(u) + c$	$\int \cosh(u) du = \sinh(u) + c$
5	$\int \sec^2(u) du = \tan(u) + c$	$\int \sec h^2(u) du = \tanh(u) + c$
6	$\int \csc^2(u) du = -\cot(u) + c$	$\int \csc h^2(u) du = -\coth(u) + c$
7	$\int \sec(u)\tan(u) du = \sec(u) + c$	$\int \sec h(u) \tanh(u) du = -\sec h(u) + c$
8	$\int \csc(u)\cot(u) du = -\csc(u) + c$	$\int \csc h(u) \coth(u) du = -\csc h(u) + c$
9	$\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2 + a^2}} = \sinh^{-1}(\frac{u}{a}) + c$
10	$\int \frac{-du}{\sqrt{a^2 - u^2}} = \cos^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{\sqrt{u^2 - a^2}} = \cosh^{-1}(\frac{u}{a}) + c$
11	$\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{a^2 - u^2} = \begin{cases} \frac{1}{a}\tanh^{-1}(\frac{u}{a}) + c & \lvert u \rvert \prec a \\ \frac{1}{a}\coth^{-1}(\frac{u}{a}) + c & \lvert u \rvert \succ a \end{cases}$
12	$\int \frac{-du}{a^2 + u^2} = \frac{1}{a}\cot^{-1}(\frac{u}{a}) + c$	
13	$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\sec h^{-1}(\frac{u}{a}) + c$
14	$\int \frac{-du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\csc^{-1}(\frac{u}{a}) + c$	$\int \frac{du}{u\sqrt{a^2 + u^2}} = \frac{1}{a}\csc h^{-1}(\frac{u}{a}) + c$

Method [1]

Integration By Substitution

The goal of this method is to transform the integral into a standard from

To evaluate the integral $I=\int f[g(x)]\,g'(x)dx$ **carry out the following steps**

1- substitute $u=g(x)$ **, then** $du=g'(x)dx$ **to obtain** $I=\int f(u)du$

2- Evaluate $I=\int f(u)du$ **by integrating w.r.t** u

3- Replace u **by** $g(x)$ **in the final result**

Evaluate $I=\int \frac{dx}{\sqrt[3]{1-2x}}$

Solution :- $I=\int (1-2x)^{-\frac{1}{3}}dx$ **Let** $u=1-2x \Rightarrow du=-2dx \Rightarrow dx=\frac{du}{-2}$

$$\therefore\ I=\int (1-2x)^{-\frac{1}{3}}dx \Rightarrow I=\int u^{-\frac{1}{3}}\frac{du}{-2}=\frac{-1}{2}\int u^{-\frac{1}{3}}du=\frac{-1}{2}\frac{u^{\frac{2}{3}}}{\frac{2}{3}}+c=\frac{-3}{4}(1-2x)^{\frac{2}{3}}+c$$

Evaluate $I=\int \sin^2(5x)\cos(5x)dx$

Solution :- **Let** $u=\sin(5x) \Rightarrow du=5\cos(5x)dx \Rightarrow dx=\frac{du}{5\cos(5x)}$

$$\therefore\ I=\int \sin^2(5x)\cos(5x)dx \Rightarrow I=\int u^2\cancel{\cos(5x)}\frac{du}{5\cancel{\cos(5x)}}=\frac{1}{5}\int u^2du=\frac{1}{5}\frac{u^3}{3}+c=\frac{1}{15}[\sin(5x)]^3+c$$

Evaluate $I=\int xe^{x^2+1}dx$

Solution :- **Let** $u=x^2+1 \Rightarrow du=2xdx \Rightarrow dx=\frac{du}{2x}$

$$\therefore\ I=\int xe^{x^2+1}dx \Rightarrow I=\int xe^u\frac{du}{2x}=\frac{1}{2}\int e^u du=\frac{1}{2}e^u+c=\frac{1}{2}e^{x^2+1}+c$$

Evaluate $I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin(3x)}dx$

Solution :- **Let** $u=4+\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\frac{du}{3\cos(3x)}$

$$I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin(3x)}dx \Rightarrow I=\frac{1}{3}\int\frac{3\cancel{\cos(3x)}}{u}\frac{du}{3\cancel{\cos(3x)}}=\frac{1}{3}\int\frac{1}{u}du=\ln(u)+c=\frac{1}{3}\ln[4+\sin(3x)]+c$$

Evaluate $I=\frac{1}{3}\int\frac{3\cos(3x)}{4+\sin^2(3x)}dx$

Solution :- **Let** $u=\sin(3x) \Rightarrow du=3\cos(3x)dx \Rightarrow dx=\frac{du}{3\cos(3x)}$

$$\Rightarrow I=\frac{1}{3}\int\frac{\cancel{3\cos(3x)}}{2^2+u^2}\frac{du}{\cancel{3\cos(3x)}}=\frac{1}{3}\int\frac{1}{2^2+u^2}du=\frac{1}{3}\frac{1}{a}\tan^{-1}(\frac{u}{a})+c=\frac{1}{3}\frac{1}{2}\tan^{-1}(\frac{u}{2})+c$$

Evaluate $I=\int\frac{dx}{\sqrt{4-9x^2}}$

Solution :- $I=\int\frac{dx}{\sqrt{4-(3x)^2}}$ **Let** $u=3x \Rightarrow du=3dx \Rightarrow dx=\frac{du}{3}$

$$\Rightarrow I=\int\frac{1}{\sqrt{2^2-u^2}}\frac{du}{3}=\frac{1}{3}\int\frac{1}{\sqrt{a^2-u^2}}du=\frac{1}{3}\sin^{-1}(\frac{u}{a})+c=\frac{1}{3}\sin^{-1}(\frac{u}{2})+c=\frac{1}{3}\sin^{-1}(\frac{3x}{2})+c$$

Evaluate $I=\int\frac{\cos(x)dx}{\sin^2(x)}$ **Let** $u=\sin(x) \Rightarrow du=\cos(x)dx \Rightarrow dx=\frac{du}{\cos(x)}$

Solution $I=\int\frac{\cos(x)dx}{\sin^2(x)}=\int\frac{\cancel{\cos(x)}}{u^2}\frac{du}{\cancel{\cos(x)}}=\int u^{-2}du=\frac{u^{-2+}}{-2+1}+c$

Evaluate $I=\int\tan^3(3x)3\sec^2(3x)dx$

Solution :- **Let** $u=\tan(3x) \Rightarrow du=3\sec^2(3x)dx \Rightarrow dx=\frac{du}{3\sec^2(3x)}$

$$I=\int\tan^3(3x)3\sec^2(3x)dx \Rightarrow \quad I=\int u^3 3\sec^2(3x)\frac{du}{3\sec^2(3x)}=\int u^3du=\frac{u^4}{4}+c=\frac{1}{4}[\tan(3x)]^4+c$$

Evaluate $I=\int \frac{\sin^2(2x)}{1+\cos(2x)}dx$

Solution :- $I=\int \frac{\sin^2(2x)}{1+\cos(2x)}dx=\int \frac{1-\cos^2(2x)}{1+\cos(2x)}dx=\int \frac{(1-\cos 2x)\cancel{(1+\cos 2x)}}{\cancel{1+\cos(2x)}}$

$$\Rightarrow \int [1-\cos(2x)]dx = x-\frac{1}{2}\sin(2x)+c$$

Evaluate $I=\int \frac{\sqrt{x}}{4+x^3}dx$

Solution :- $I=\int \frac{\sqrt{x}}{4+\left((x)^{\frac{3}{2}}\right)^2}dx$ **Let** $u=x^{\frac{3}{2}} \Rightarrow du=\frac{3}{2}x^{\frac{1}{2}}dx=\frac{3}{2}\sqrt{x}\ dx$

$$\Rightarrow dx=\frac{du}{\frac{3}{2}\sqrt{x}} \Rightarrow \int \frac{\sqrt{x}}{2^2+(u)^2}\frac{du}{\frac{3}{2}\sqrt{x}}=\frac{2}{3}\int \frac{du}{a^2+(u)^2}=\frac{2}{3}\frac{1}{2}\tan^{-1}(\frac{u}{2})=\frac{1}{3}\tan^{-1}(\frac{x^{3/2}}{2})+c$$

Evaluate $I=\int \sec^2(x)dx$

Solution :- $\Rightarrow I=\int \sec^2(x)dx=\tan(x)+c$

Evaluate $I=\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx$

Solution :- **Let** $u=\sin^{-1}(x) \Rightarrow du=\frac{1}{\sqrt{1-x^2}}dx \Rightarrow dx=\sqrt{1-x^2}du$

$$I=\int \frac{\sin^{-1}(x)}{\sqrt{1-x^2}}dx=\int \frac{u}{\sqrt{1-x^2}}\sqrt{1-x^2}du \Rightarrow I\int udu=\frac{u^2}{2}+c=\frac{[\sin^{-1}(x)]^2}{2}+c$$

Evaluate $I = \int \frac{e^x}{1+e^{2x}}dx \Rightarrow \int \frac{e^x}{1+(e^x)^2}dx$

Solution :- **Let** $u = e^x \Rightarrow du = e^x dx \Rightarrow dx = \frac{du}{e^x}$

$$I = \int \frac{e^x}{1+(e^x)^2}dx = \int \frac{\cancel{e^x}}{1+(u)^2}\frac{du}{\cancel{e^x}} \Rightarrow I = \int \frac{du}{1+u^2} = \tan^{-1}(u)+c = \tan^{-1}(e^x)+c$$

Evaluate $I = \int \frac{[\ln(x)]^2}{x}dx$

Solution :- **Let** $u = \ln(x) \Rightarrow du = \frac{1}{x}dx \Rightarrow dx = xdu$

$$I = \int \frac{[u]^2}{x}xdx = \int u^2 du \Rightarrow I = \frac{u^3}{3} = \frac{[\ln(x)]^3}{3} + c$$

H.W : Find the integrations :

1	$\int (x-\frac{1}{x})^2$	$\int x.2^{x^2+3}dx$	$\int \frac{\sec^2(x)}{1+\tan^2(x)}dx$	$\int \frac{e^x}{1+e^{2x}}dx$
2	$\int \tan^2(3x)dx$	$\int \tan(4x)dx$	$\int \frac{dx}{x[1+(\ln(x))^2]}$	$\int \frac{[\ln(x)]^3}{x}$
3	$\int \frac{\sec^2[\ln(x)]}{x}dx$	$\int \frac{2\sin(\sqrt{x})}{\sqrt{x}\sec(\sqrt{x})}dx$	$\int \frac{dx}{\sqrt{x}(1+x)}$	$\int \sin^2(x)dx$
4	$\int_4^9 \frac{dx}{x-\sqrt{x}}$	$\int_0^{\pi} \sin^3(x)\cos(x)dx$	$\int_0^{\infty} e^{-x-e^{-x}}dx$	$\int \tan(x)\frac{\ln[\cos(x)]}{2}dx$