Method [3]

Integration of power trigonometric function :

Consider the following integrals forms

A	$\int \sin^m(u)\cos^n(u)\,du$	$\int \sinh^m(u)\cosh^n(u)\,du$
B	$\int \tan^m(u)\sec^n(u)\,du$	$\int \tanh^m(u)\operatorname{sec}h^n(u)\,du$
C	$\int \cot^m(u)\csc^n(u)\,du$	$\int \coth^m(u)\operatorname{csc}h^n(u)\,du$

Under Type (A) There are three cases :

Case (1)

If (m) is odd , We factor out $[\sin(u)\ \sinh(u)]$ **and change the remaining even power of** $[\sin(u)\ \sinh(u)]$ **to** $[\cos(u)\ \cosh(u)]$ **using :**

$$\sin^2(u) = 1-\cos^2(u) \qquad , \qquad \sinh^2(u) = \cosh^2(u)-1$$

Evaluate $I = \int \sin^5(2x)\cos^{\frac{-3}{2}}(2x)\,dx$

Solution :- $\Rightarrow I = \int \sin^4(2x)\cos^{\frac{-3}{2}}(2x)\ \sin(2x)dx$

$$\Rightarrow I = \int \left(1-\cos^2(2x)\right)^2 \cos^{\frac{-3}{2}}(2x)\ \ \sin(2x)\,dx = \int \left(1-2\cos^2(2x)+\cos^4(2x)\right)\cos^{\frac{-3}{2}}(2x)\ \sin(2x)\,dx$$

$$\Rightarrow \int \left(\cos^{\frac{-3}{2}}(2x) - 2\cos^{\frac{1}{2}}(2x) + \cos^{\frac{5}{2}}(2x)\right)\sin(2x)dx$$

$$\Rightarrow \int \left(\cos^{\frac{-3}{2}}(2x)\sin(2x) - 2\cos^{\frac{1}{2}}(2x)\sin(2x) + \cos^{\frac{5}{2}}(2x)\ \sin(2x)\right)dx$$

$$\Rightarrow \quad = -\frac{1}{2}\frac{\cos^{\frac{-1}{2}}(2x)}{\frac{-1}{2}} + \frac{\cos^{\frac{3}{2}}(2x)}{\frac{3}{2}} - \frac{1}{2}\frac{\cos^{\frac{7}{2}}(2x)}{\frac{7}{2}} + c$$

Case (2)

If (n) is odd , We factor out $[\cos(u)\cosh(u)]$ and change the remaining even power of $[\cos(u)\cosh(u)]$ to $[\sin(u)\sinh(u)]$ using :

$$\cos^2(u) = 1-\sin^2(u) \qquad , \qquad \cosh^2(u) = 1+\sinh^2(u)$$

Evaluate $I = \int \sin^4(3x)\cos^3(3x)\,dx$

Solution :- $\Rightarrow I = \int \cos^2(3x)\,\sin^4(3x)\,\cos(3x)dx = \int \left(1-\sin^2(3x)\right)\sin^4(3x)\,\cos(3x)dx$

$$\Rightarrow = \int \left(\sin^4(3x)-\sin^6(3x)\right)\cos(3x)dx = \int \sin^4(3x)\cos(3x)dx - \int \sin^6(3x)\,\cos(3x)dx$$

$$= \frac{1}{3}\frac{\sin^5(3x)}{5} - \frac{1}{3}\frac{\sin^7(3x)}{7}$$

Case (3)

If both (n) and (m) are even ,we reduce the degree of the expression by using :

$$\sin^2(u) = \frac{1-\cos(2u)}{2} \qquad , \qquad \sinh^2(u) = \frac{\cosh(2u)-1}{2}$$

$$\cos^2(u) = \frac{1+\cos(2u)}{2} \qquad , \qquad \cosh^2(u) = \frac{\cosh(2u)+1}{2}$$

Evaluate $I = \int \sin^2(2x)\cos^2(2x)dx$

Solution :- $\Rightarrow \frac{1}{4}\int (1-\cos(4x))(1+\cos(4x))dx = \frac{1}{4}\int \left(1-\cos^2(4x)\right)dx$

$$\Rightarrow \frac{1}{4}\int \left(1-\frac{1+\cos(8x)}{2}\right)dx = \frac{1}{4}\int \left(\frac{1}{2}-\frac{1}{2}\cos(8x)\right)dx = \frac{1}{4}\left[\frac{x}{2}-\frac{1}{16}\sin(8x)\right]+c$$

Under Type (B) There are two cases :

Case (1)

If (n) is even , We factor out $\left[\sec^2(u)\,\sec^2 h(u)\right]$ **and change the remaining even power of** $\left[\sec(u)\,\sec h(u)\right]$ **to** $\left[\tan(u)\,\tanh(u)\right]$ **using :**

$$\sec^2(u) = 1+\tan^2(u) \quad , \quad \sec h^2(u) = 1-\tanh^2(u)$$

Evaluate $I = \int \tan^{-\frac{1}{3}}(x)\,\sec^4(x)\,dx$

Solution :- $\Rightarrow \int \sec^2(x)\tan^{-\frac{1}{3}}(x)\ \sec^2(x)\,dx = \int \left(1+\tan^2(x)\right)\tan^{-\frac{1}{3}}(x)\sec^2(x)dx$

$$\Rightarrow \int\left(\tan^{-\frac{1}{3}}(x)+\tan^{\frac{5}{3}}(x)\right)\sec^2(x)\,dx = \left[\frac{\tan^{\frac{2}{3}}(x)}{2/3}+\frac{\tan^{\frac{8}{3}}(x)}{8/3}\right]+c$$

Case (2)

If (m) is odd , We factor out $\left[\sec(u)\tan(u)\{\sec h(u)\tan(u)\}\right]$ **and change the remaining even power of** $\left[\sec(u)\,\sec h(u)\right]$ **to** $\left[\tan(u)\,\tanh(u)\right]$ **using :**

$$\tan^2(u) = \sec^2(u)-1 \quad , \quad \tanh^2(u) = 1-\sec h^2(u)$$

Evaluate $I = \int \tan^3(2x)\sec^{-\frac{1}{4}}(2x)\,dx$

Solution :- $I = \int\left(\sec^2(2x)-1\right)\ \sec^{-\frac{5}{4}}(2x)\ \ \sec(2x)\tan(2x)\,dx$

$$\Rightarrow \int\left(\sec^{\frac{3}{4}}(2x)-\sec^{-\frac{5}{4}}(2x)\right)\sec(2x)\tan(2x)\,dx = \frac{1}{2}\left[\frac{\sec^{\frac{7}{4}}(2x)}{7/4}-\frac{\sec^{-\frac{1}{4}(2x)}}{-1/4}\right]+c$$

Under Type (C) There are two cases similar to there of type (B) where :

$$\csc^2(u) = \cot^2(u) + 1 \quad , \quad \csc h^2(u) = \coth^2(u) - 1$$

Evaluate $I = \int \cot^3(x)\csc^4(x)\,dx$

Solution :-

$$I = \int \csc^2(x)\ \cot^3(x)\ \csc^2(x)\ dx$$
$$\Rightarrow \int (\cot^2(x)+1)\cot^3(x)\csc^2(x)\,dx$$
$$\Rightarrow \int \left(\cot^5(x)\ \csc^2(x) + \cot^3(x)\ \csc^2(x)\right)\ dx$$
$$= \frac{-\cot^6(x)}{6} + \frac{-\cot^4(x)}{4}$$

H.W

$\int \sin^5(2x)\,dx$	$\int \csc^2(x)\,dx$	$\int \cos^2(x)\,dx$	$\int \cot^4(3x)dx$
$\int \tan^3(x)\sec(x)\,dx$	$\int \cot^3(2x)\csc^4(2x)dx$	$\int \sin^3(2x)\,dx$	$\int \cos^3(x)\sin^{\frac{1}{2}}(x)\,dx$
$\int \sin^2(x)\cos^2(x)\,dx$	$\int \tan(\frac{x}{2})\sec^2(\frac{x}{2})\,dx$	$\int \sin^4(x)\cos^{-2}(x)\,dx$	$\int \sin(x)dx$