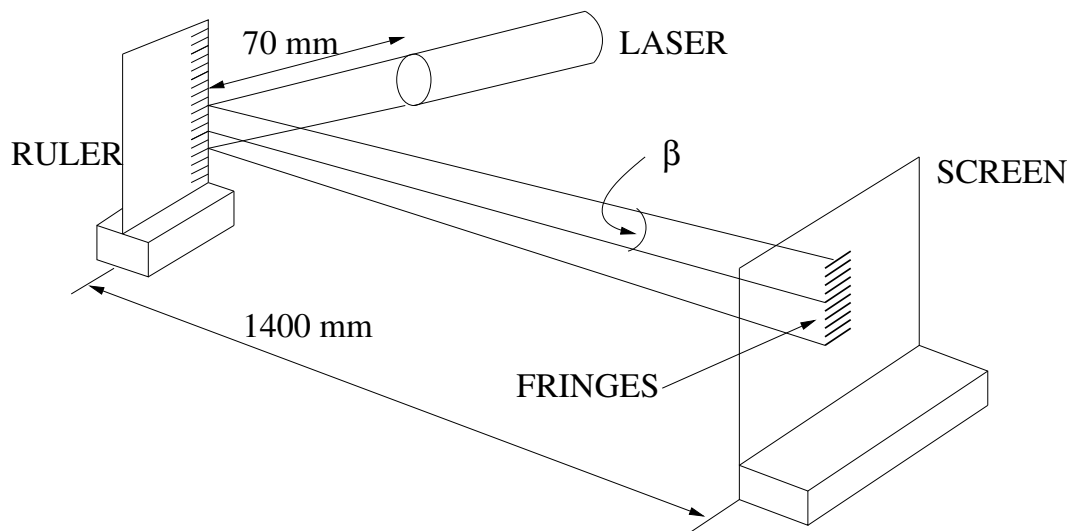


Solution to Experimental Question 2

Section 1

- i. A typical geometric layout is as shown below.
 - (a) Maximum distance from ruler to screen is advised to increase the spread of the diffraction pattern.
 - (b) Note that the grating (ruler) lines are horizontal, so that diffraction is in the vertical direction.



- ii. Vis a vis the diffraction phenomenon, $\beta = \left(\frac{y}{1400 \text{ mm}} \right)$

The angle β is measured using either a protractor (not recommended) or by measuring the value of the fringe separation on the screen, y , for a given order N .

If the separation between 20 orders is measured, then $N = \pm 10$ ($N = 0$ is central zero order).

The values of y should be tabulated for $N = 10$. If students choose other orders, this is also acceptable.

| N | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 | ± 10 |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| $2y \text{ mm}$ | 39.0 | 38.5 | 39.5 | 41.0 | 37.5 | 38.0 | 39.0 | 38.0 | 37.0 | 37.5 |
| $y \text{ mm}$ | 19.5 | 19.25 | 19.75 | 20.5 | 18.75 | 19.0 | 19.5 | 19.0 | 18.5 | 18.75 |

Mean Value = $(19.25 \pm 1.25) \text{ mm}$

i.e. Mean “spot” distance = 19.25 mm for order $N = 10$.

From observation of the ruler itself, the grating period, $h = (0.50 \pm 0.02) \text{ mm}$.

Thus in the relation

$$\begin{aligned}
 N\lambda &= \pm h \sin \beta \\
 N &= 10 \\
 h &= 0.5 \text{ mm} \\
 \sin \beta \simeq \beta &= \frac{y}{1400 \text{ mm}} = 0.01375 \\
 10\lambda &= 0.006875 \text{ mm} \\
 \lambda &= 0.0006875 \text{ mm}
 \end{aligned}$$

Since β is small, $\frac{\delta \lambda}{\lambda} \simeq \frac{\delta h}{h} + \frac{\delta y}{y} \simeq 10\%$

i.e. measured $\lambda = (690 \pm 70) \text{ nm}$

The accepted value is 680 nm so that the departure from accepted value equals 1.5%.

Section 2

This section tests the student's ability to make semi-quantitative measurements and the use of judgement in making observations.

- i. Using the $T = 50\%$ transmission disc, students should note that the transmission through the tank is greater than this value. Using a linear approximation, 75% could well be estimated. Using the hint about the eye's logarithmic response, the transmission through the tank could be estimated to be as high as 85%.

Any figure for transmission between 75% and 85% is acceptable.

- ii. Calculation of the transmission through the tank, using

$$T = 1 - R = 1 - \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2$$

for each of the four surfaces of the tank, and assuming $n = 1.59$ for the perspex, results in a total transmission

$$T_{\text{total}} = 80.80\%$$

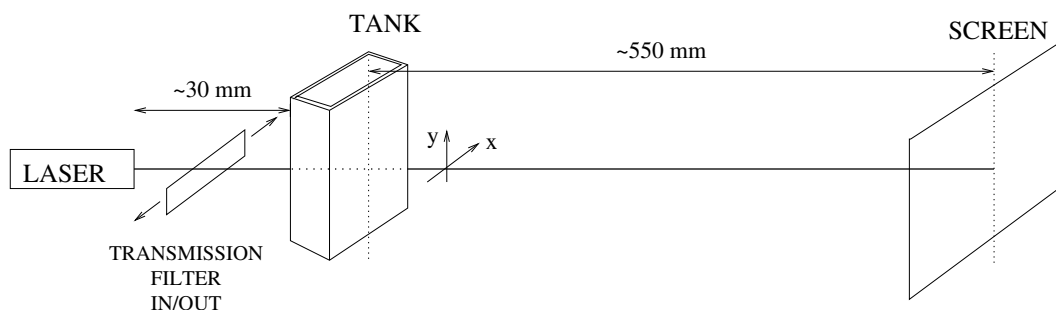
Section 3

With water in the tank, surfaces 2 and 3 become perspex/water interfaces instead of perspex/air interfaces, as in (ii).

The resultant value is

$$T_{\text{total}} = 88.5\%$$

Section 4

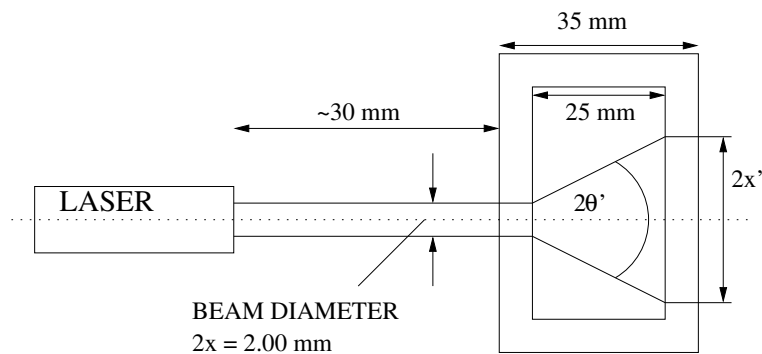


Possible configuration for section 4 (and sections 2 and 3)

With pure water in the tank only, we see from Section 3 that the transmission T is

$$T_{\text{Water}} \simeq 88\%$$

The aim here is to determine the beam divergence (scatter) and transmission as a function of milk concentration. Looking down on the tank, one sees



- i. The entrance beam diameter is 2.00 mm. The following is an example of the calculations expected:
With 0.5 mL milk added to the 50 mL water, we find

$$\text{Scatterer concentration} = \frac{0.5}{50} = 1\% = 0.01$$

Scattering angle

$$2x' = 2.2 \text{ mm} \quad ; \quad 2\theta' = \frac{2x'}{30} = 0.073$$

Transmission estimated with the assistance of the neutral density filters

$$T_{\text{total}} = 0.7 \quad .$$

Hence

$$T_{\text{milk}} = \frac{0.7}{0.88} = 0.79$$

Note that

$$T_{\text{milk}} = \frac{T_{\text{total}}}{T_{\text{water}}} \quad \text{and} \quad T_{\text{water}} = 0.88 \quad (1)$$

If students miss the relationship (1), deduct one mark.

- ii. & iii. One thus obtains the following table of results. $2\theta'$ can be determined as shown above, OR by looking down onto the tank and using the protractor to measure the value of $2\theta'$. It is important to note that even in the presence of scattering, there is still a direct beam being transmitted. It is much stronger than the scattered radiation intensity, and some skill will be required in measuring the scattering angle $2\theta'$ using either method. Making the correct observations requires observational judgement on the part of the student.

Typical results are as follows:

| | | | | | | | | | |
|----------------------|----------|------|------|------|------|------------|------|------|------|
| Milk volume (mL) | 0 | 0.5 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 |
| % Concentration | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| $2x'$ | 2.00 | 2.2 | 6.2 | 9.4 | 12 | Protractor | | | |
| $2\theta'$ (Degrees) | ~ 0 | 4 | 12 | 18 | 23 | 28 | 36 | 41 | 48 |
| T_{milk} | 1.0 | 0.79 | 0.45 | 0.22 | 0.15 | 0.12 | 0.08 | 0.06 | 0.05 |

- iii. From the graphed results in Figure 1, one obtains an approximately linear relationship between milk concentration, C , and scattering angle, $2\theta'$ ($= \phi$) of the form

$$\phi = 6C \quad .$$

- iv. Assuming the given relation

$$I = I_0 e^{-\mu z} = T_{\text{milk}} I_0$$

where z is the distance into the tank containing milk/water.

We have

$$T_{\text{milk}} = e^{-\mu z}$$

Thus

$$\ln T_{\text{milk}} = -\mu z \quad , \text{ and } \mu = \text{constant} \times C$$

Hence $\ln T_{\text{milk}} = -\alpha z C$.

Since z is a constant in this experiment, a plot of $\ln T_{\text{milk}}$ as a function of C should yield a straight line. Typical data for such a plot are as follows:

| | | | | | | | | | |
|-----------------------|-----|-------|------|-------|-------|-------|-------|-------|-------|
| % Concentration | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| T_{milk} | 1.0 | 0.79 | 0.45 | 0.22 | 0.15 | 0.12 | 0.08 | 0.06 | 0.05 |
| $\ln T_{\text{milk}}$ | 0 | -0.24 | -0.8 | -1.51 | -1.90 | -2.12 | -2.53 | -2.81 | -3.00 |

An approximately linear relationship is obtained, as shown in Figure 2, between $\ln T_{\text{milk}}$ and C , the concentration viz.

$$\ln T_{\text{milk}} \simeq -0.4C = -\mu z$$

Thus we can write

$$T_{\text{milk}} = e^{-0.4C} = e^{-\mu z}$$

For the tank used, $z = 25 \text{ mm}$ and thus

$$0.4C = 25\mu \quad \text{or} \quad \mu = 0.016C \quad \text{whence} \quad \alpha = 0.016 \text{ mm}^{-1}\%^{-1}$$

By extrapolation of the graph of $\ln T_{\text{milk}}$ versus concentration C , one finds that for a scatterer concentration of 10%

$$\mu = 0.160 \text{ mm}^{-1} \quad .$$

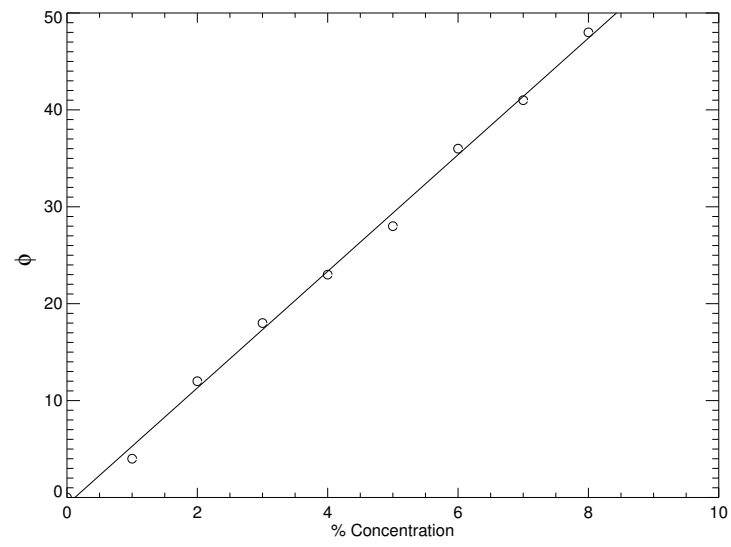


Figure 1: Sample plot

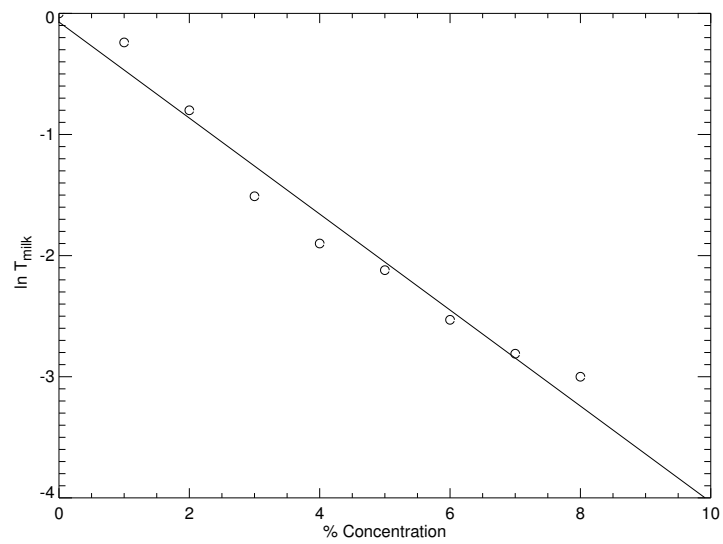


Figure 2: Sample plot

Detailed Mark Allocation

Section 1

| | |
|--|-----|
| A clear diagram illustrating geometry used with appropriate allocations | [1] |
| Optimal geometry used - as per model solution (laser close to ruler) | [1] |
| Multiple measurements made to ascertain errors involved | [1] |
| Correctly tabulated results | [1] |
| Sources of error including suggestion of ruler variation (suggested by non-ideal diffraction pattern) | [1] |
| Calculation of uncertainty | [1] |
| Final result | [2] |
| Allocated as per: | |
| $\pm 10\%$ (612, 748 nm) | [2] |
| $\pm 20\%$ (544, 816 nm) | [1] |
| \pm anything worse | [0] |

Section 2

| | |
|--|-----|
| For evidence of practical determination of transmission rather than simply “back calculating”. Practical range 70 – 90% | [1] |
| For correct calculation of transmission (no more than 3 significant figures stated) | [1] |

Section 3

| | |
|--|-----|
| Correct calculation with no more than 3 significant figures stated and an indication that the measurement was performed | [1] |
|--|-----|

Section 4

| | |
|--|-----|
| Illustrative diagram including viewing geometry used, i.e. horizontal/vertical | [1] |
| For recognizing the difference between scattered light and the straight-through beam | [1] |
| For taking the T_{water} into account when calculating T_{milk} | [1] |
| Correctly calculated and tabulated results of T_{milk} with results within 20% of model solution | [1] |
| Using a graphical technique for determining the relationship between scatter angle and milk concentration | [1] |
| Using a graphical technique to extrapolate T_{milk} to 10% concentration | [1] |
| Final result for μ | [2] |
| Allocated as $\pm 40\%$ [2], $\pm 60\%$ [1], anything worse [0] | |
| A reasonable attempt to consider uncertainties | [1] |
| TOTAL | 20 |