Measuring the Change in the Refractive Index of Water with a Michelson Interferometer

Andy Tan
Science One 2009-2010
The University of British Columbia
Vancouver, Canada

May 5th, 2010

Measuring the Change in the Refractive Index of Water with a Michelson Interferometer

Andy Tan

The refractive index *n* of a substance varies as a function of temperature. A water-containing cell was placed in one leg of a Michelson interferometer, producing fringe shifts (laser interference patterns) on a screen as the water cooled. By counting the fringe shifts for each change in temperature of 1°C, a set of values for *n* could be created. These values agreed with values of *n* found in two separate papers to within 2×10^{-5}.

Introduction

When hot water is poured into a glass containing cold water, there appears to be "ripples" within the glass. It seems like two immiscible liquids have been put together and two liquid phases can almost be distinguished. This is due to the difference in the refractive index of water at different temperatures.

A Michelson interferometer (Figure 1) can be used to measure changes in the index of refraction by inserting a water-containing cell in one leg of the interferometer and letting the laser beam pass through the water. As the laser beam interferes with another beam from a second leg of the interferometer, an image consisting of a pattern of stripes (light and dark lines) appears on a screen placed in front of the laser. If the temperature of the water changes, then so does the refractive index of the water, which affects the manner in which laser beams from each leg of the interferometer interfere. This is seen as movement of the pattern of light and dark stripes on the screen.

The object of this paper is to measure the change in the refractive index of water by counting the number of dark-light-dark fringe shifts that appeared per degree Celsius.

Diagram of Michelson Interferometer

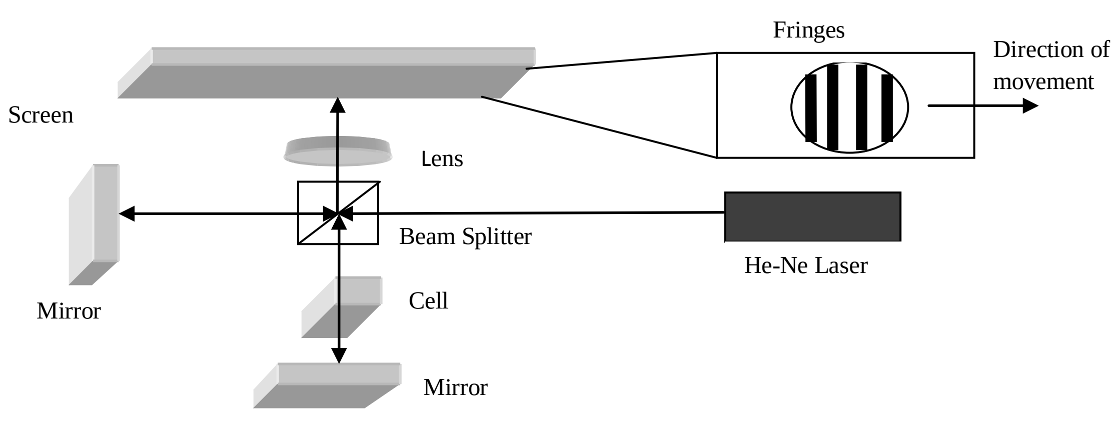

Figure 1. Diagram of the Michelson interferometer. The distance between the beam splitter to the screen and each mirror is 15 cm, the distance between the beam splitter and the laser is 30 cm.

Methods

A Michelson interferometer was first constructed from a 5mW 632.8 nm He-Ne laser, a beam splitter, two mirrors, a converging lens and a white screen (Figure 1). The mirrors and beam splitter were adjusted as necessary to observe an interference pattern of light and dark fringes on the display screen. A lens was placed before the screen to widen the fringe pattern and facilitate observation.

A rectangular optical cell was then constructed out of three flat pieces of wood and two microscope slides (Figure 2) and placed in one leg of the Michelson interferometer (Figure 1). The cell was filled with warm, deionized water at about 70°C and allowed to cool to room temperature. A thermometer supported by a stand was suspended in the cell as close to the laser beam as possible without disturbing it.

Diagram of the Glass Cell

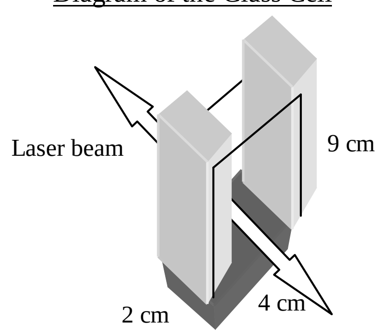

Figure 2. The base and the two long vertical sides were made by joining the pieces of wood (pine) with a framing machine, while the front and back faces of the cell were made by gluing one microscope slide on each face. The top of the cell was left open.

As the water cooled, the refractive index of the water changed, affecting the way light from one leg of the interferometer interfered with light from the other leg (e.g. from constructive, to destructive, to constructive again) causing fringe shifts, which move in a direction perpendicular to the stripes of light, to appear on the screen. At any particular point on the screen, one fringe shift is said to have occurred if a dark stripe passed by that point, followed by a light stripe, followed by a dark stripe again. The number of fringe shifts passing a marked point on the screen was recorded for every drop of a degree in temperature. One cycle of constructive-destructive-constructive interference (one fringe shift) occurs as the number of wavelengths of light contained in the cell changes by 1.

The total path length of the laser as it passes through the cell twice is $2L$ where L is the interior length of the cell. The wavelength of the laser in the cell λ is given by

$$\lambda = \frac{\lambda_{vac}}{n} \quad (1)$$

where n is the index of refraction of the cell's contents. Then the number of wavelengths m in the cell is

$$m = \frac{2L}{\lambda} = \frac{2Ln}{\lambda_{vac}} \quad (2)$$

As the number of wavelengths changes by 1, one fringe shift is seen and so the number of fringe shifts seen is equal to the difference in the number of wavelengths at temperatures T_1 and T_2. The difference in the number of wavelengths at two temperatures T_1 and T_2 is given by

$$\Delta m = m_2 - m_1 \quad (3)$$

But since n changes with temperature, m_2 and m_1 are given by

$$m_2 = \frac{2Ln_2}{\lambda_{vac}} \quad \text{and} \quad m_1 = \frac{2Ln_1}{\lambda_{vac}}. \tag{4), (5}$$

Combining equations (3), (4), (5) and rearranging for n_2 - n_1 results in the following expression

$$n_2 - n_1 = \frac{\Delta m\, \lambda_{vac}}{2L} \tag{6}$$

The change in the index of refraction can be written as $\Delta n = n_2 - n_1$. Making this substitution and dividing both sides of (6) by $\Delta T = T_2 - T_1$ yields the final expression

$$\frac{\Delta n}{\Delta T} = \frac{\Delta m\, \lambda_{vac}}{2L\Delta T} \tag{7}$$

Δm, the number of fringe shifts seen for each change in temperature of 1°C was observed and recorded. $\frac{\Delta n}{\Delta T}$ for a particular temperature could then be calculated. This process was repeated about 5 times for each temperature from 28°C to 72°C and the average $\frac{\Delta n}{\Delta T}$ found for each temperature was plotted (Figure 3).

Results & Discussion

Table 1 contains the errors that arose when conducting the measurements.

Summary of possible errors

Source of error	Error contribution
Fringe miscount at $T \geq 60$°C	6.7%; 1 miscount in 15 fringe shifts
Fringe miscount at 45°C $\leq T <$ 60°C	2.5%; 1 miscount in 40 fringe shifts
Fringe miscount at $T <$ 45°C	1%; 1 miscount in 100 fringe shifts
Cell length measurement	0.1%
Cell length thermal expansion	0.0225%

Table 1. The error rate is greater at higher temperatures due to the greater rate of cooling, and hence a greater speed at which fringe shifts occurred. The cell length was measured by a digital caliper with an accuracy of 0.02 mm. The cell length expansion for pine is 5 parts per million per change in °C measured along the grain[1]. With a cell length of 2 cm and maximum temperature difference of 45°C, the change in length of the cell is at most 0.0225%.

The length of the cell was measured prior to each trial to minimize the effect of the swelling of wood due to water. The fringe miscounts were chiefly due to the necessity of monitoring the thermometer while counting fringes. There were other factors that limited the quality of the data such as the precision of the thermometer, the warming of the air by the faces of the cell, and the exact wavelength of the laser output. Their contribution to error was difficult to quantify.

It was found that as the temperature increased, the rate of change of the refractive index of water grew increasingly negative (Figure 3). As $\frac{\Delta n}{\Delta T}$ appears to be nonlinear in temperature, n is likely nonlinear in temperature as well. There appears to be anomalies in the temperature ranges of 33°C to 39°C and 50°C to 57°C which deviate from the general decreasing trend. There were fewer trials conducted in the lower temperature range of 40°C and below, which may have lead to rougher data in comparison to temperatures above 40°C. At temperatures 50°C to 57°C, the data is again irregular as this was the range where the refractive index of water had changed sufficiently compared to at 72°C that the beams from each leg of the interferometer strayed from alignment and no longer interfered with each other (perfectly positioning the mirrors right at the start of the trial to avoid this problem was difficult for practical reasons). A readjustment of the mirrors became necessary to continue getting interference of the laser.

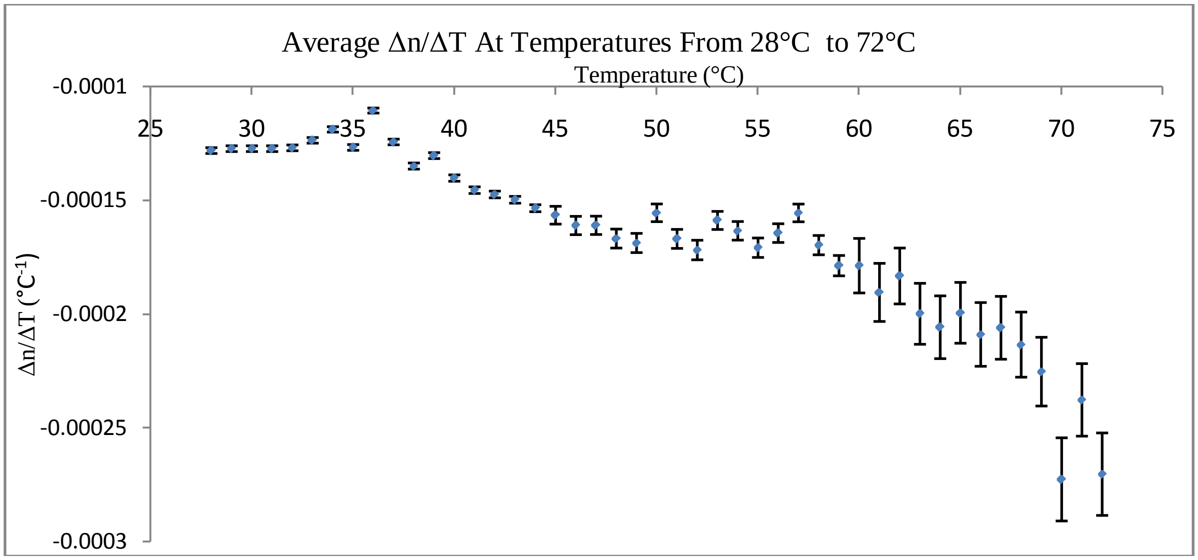

Figure 3. Each data point is the average value of $\frac{\Delta n}{\Delta T}$ for a particular temperature as calculated by equation (7) from several trials, and is interpreted as the change in *n* from the temperature indicated to the temperature one degree lower. The error bars are a percentage of each $\frac{\Delta n}{\Delta T}$ value as indicated in Table 1.

In order to compare the results of $\frac{\Delta n}{\Delta T}$ with values of *n* found in the literature, the value of *n* at 30°C was set to 1.33107 (the average of values of *n* at 30°C published by Schiebener *et al*. [2] and Stanley [3]). The negative values shown in Figure 3 were added cumulatively one at a time to 1.33107, and each sum represented the value of *n* at a particular temperature. Thus, a set of values of *n* was calculated from the measured values of $\frac{\Delta n}{\Delta T}$ by first setting *n* at 30°C to a standard literature value (1.33107), and then adding $\frac{\Delta n}{\Delta T}$ to 1.33107 for each degree Celsius up to 70°C. Comparisons were made between papers which published values of *n* at 1 atm for a wavelength of 632.8 nm (Figure 4).

The error ε for the purpose of comparing values of *n* (cumulative sums of Δ*n*) to values found in the literature is given by

$$\varepsilon = \sqrt{\sum_{T=30}^{72} (\varepsilon_T)^2} = 5.5 * 10^{-5} \quad (8)$$

where ε_T is the error associated with each individual measurement of the change of refractive index at temperature *T*. This method of adding errors in quadrature assumes that erring on either side of the true value is equally likely, so that an error given by the cumulative sum of the errors in Figure 3 is unnecessarily large [4]. The smaller value ε is a better indication of the error of each point presented by this paper in Figure 4.

The values obtained for $\frac{\Delta n}{\Delta T}$ yield a reasonable set of values for *n*, and the error bars of values at the same temperature overlap or are at most 2 x 10^{-5} from overlapping with two independent sources that also measured *n* at 1 atm for 632.8 nm light. The zoomed-in graphs in Figure 4 show that the values of *n* calculated by this paper are consistently larger than the other two sources. This may be due to fringe miscounts.

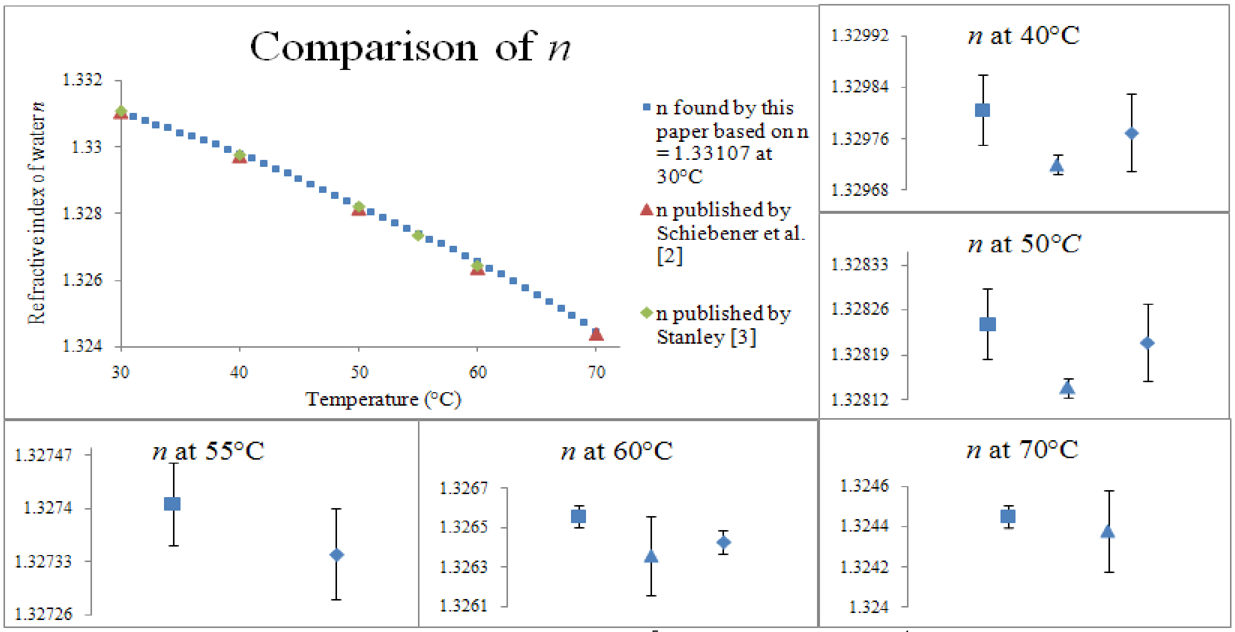

Figure 4. The error for values by Schiebener et al. [2] is 1.5 x 10^{-5} for T < 60°C and 2 x 10^{-4} for T ≥ 60°C. The error for values by Stanley [3] is 6 x 10^{-5}. The error for values calculated by this paper is 5.5 x 10^{-5}. The large graph shows the overall trend of *n*, while the smaller graphs are zoomed in on single specific temperatures so that error bars can be compared. Schiebener *et al.*[2] did not publish a value at 55°C, and Stanley [3] did not publish a value at 70°C.

Conclusion

Using a self-assembled Michelson interferometer and a home-built optical cell, the change in the refractive index of water was measured from degree to degree in temperature. The range of temperatures observed was 28°C to 72°C. By counting fringe shifts and applying that number to equation (7), a set of values for *n* could be generated with an initial value of *n* from a separate source. It was found that the set of values generated by the measurements conducted either agreed or were within 2 x 10^{-5} of agreeing with values found in the literature.

Acknowledgements

The author thanks Professor Mark Halpern for the use of the Michelson interferometer parts and his advice on how to produce the clearest fringe pattern. Microscope slides were graciously provided by Professor Carol Pollock. Measurements were conducted using a thermometer and thermometer stand borrowed from Science One. Without the expertise and materials provided by the author's father, the optical glass cell could not have been made.

References:

[1] Coefficients of Linear Expansion. *The Engineering Toolbox.* 2005. http://www.engineeringtoolbox.com/linear-expansion-coefficients-d_95.html Date accessed: Feb 26th 2010.

[2] Schiebener, P., Straub, J., Sengers, J.M.H.L, Gallagher J.S. Refractive index of water and steam as function of wavelength, temperature and density. *J. Phys. Chem. Ref. Data* **19**, 677 – 717 (1990).

[3] Stanley, E.M. Refractive index of pure water for wavelength of 6328 Ä at high pressure and moderate temperature. *J. Chem. Eng. Data* **16**, 454 – 457 (1971).

[4] Personal communication with Geoff Topping, TA for PHYS 109 in the Winter 2010 session, on Mar 9th 2010.

Appendix: Experimental Data

	$\Delta n/\Delta T$										
Temp	Feb 10th Trial 1	Feb 10th Trial 2	Feb 10th Trial 3	Feb 10th Trial 4	Feb 11th Trial 1	Feb 11th Trial 2	Feb 11th Trial 3	Feb 11th Trial 4	Feb 12th Trial 1	Feb 12th Trial 2	Average $\Delta n/\Delta T$
72						0.00027					0.00027
71						0.00025		0.00022			0.00024
70						0.00019	0.00041	0.00024		0.00025	0.00027
69						0.00022	0.00024	0.00021		0.00024	0.00023
68						0.00021	0.00021	0.00022		0.00022	0.00021
67				0.00022		0.00019	0.00021	0.00021			0.00021
66				0.00021		0.00021	0.00022	0.00022	0.00019		0.00021
65				0.00021		0.00019	0.00017	0.00021		0.00022	0.0002
64				0.00021		0.00021	0.00019	0.00021		0.00022	0.00021
63			0.00019	0.00021		0.00021		0.00019	0.0002		0.0002
62			0.00018	0.00018		0.00019	0.00019	0.00019	0.00017	0.00019	0.00018
61			0.00019	0.00018		0.00021	0.00019	0.00019			0.00019
60			0.00018	0.00016		0.00019	0.00017	0.00019	0.00017	0.00019	0.00018
59			0.00016	0.00018		0.00019	0.00017	0.00019	0.00019	0.00017	0.00018
58			0.00016	0.00021		0.00016	0.00016	0.00017	0.00017	0.00016	0.00017
57			0.00014			0.00013	0.00016	0.00017	0.00017	0.00016	0.00016
56			0.00016				0.00016	0.00016	0.00017	0.00017	0.00016
55				0.00019		0.00014		0.00017	0.00017	0.00017	0.00017
54				0.00016		0.00016	0.00017	0.00016	0.00016	0.00017	0.00016
53	0.00017839			0.00014		0.00013		0.00017	0.00016	0.00017	0.00016
52	0.00016217			0.00018		0.00016	0.00017	0.00019			0.00017
51	0.00019461			0.00016		0.00016	0.00017	0.00016		0.00016	0.00017
50	-			0.00018		0.00013	0.00016	0.00016	0.00016	0.00016	0.00016
49	0.00017839			0.00018	0.00024	0.00014	0.00014	0.00016	0.00016	0.00016	0.00017
48	0.00016217			0.00016	0.00021			0.00017	0.00016	0.00014	0.00017
47	0.00016217			0.00016	0.00018		0.00016	0.00016	0.00016	0.00016	0.00016
46	0.00017839			0.00016	0.00019		0.00016	0.00013	0.00016	0.00016	0.00016
45	0.00017839			0.00018	0.00016		0.00016	0.00013	0.00016	0.00014	0.00016

44	0.00017839				0.00016		0.00016	0.00011	0.00016	0.00016	0.00015
43	0.00017839			0.00018	0.00014		0.00014	0.00013	0.00014	0.00014	0.00015
42	0.00017839			0.00013	0.00013		0.00016	0.00014	0.00016	0.00014	0.00015
41	0.00017839				0.00013		0.00014	0.00014	0.00014	0.00014	0.00015
40	0.00016217				0.00014		0.00013	0.00013	0.00014	0.00014	0.00014
39	0.00016217				0.00011		0.00013		0.00013	0.00013	0.00013
38	0.00016217				0.00013		0.00013	0.00014	0.00013	0.00013	0.00014
37	0.00016217				8E-05		0.00013	0.00013	0.00013	0.00013	0.00012
36					0.00011		0.00011	0.00011	0.00011	0.00011	0.00011
35		0.00018			0.00011		0.00013	0.00013	0.00011	0.00011	0.00013
34		0.00016			0.00011		0.00011	0.00011	0.00011	0.00011	0.00012
33		0.00016			0.00011			0.00013	0.00011	0.00011	0.00012
32		0.00016						0.00011	0.00011		0.00013
31		0.00014						0.00011			0.00013
30		0.00014						0.00011			0.00013
29		0.00014						0.00011			0.00013
28		0.00013									0.00013