Lab 10: Gaussian Beam Optics

1 Introduction

In this experiment, you will observe the Gaussian properties of laser beams. The measurements will be made by partially blocking a beam with a smooth, straight edge (a razor blade) and measuring the fraction of light that is not blocked relative to the position of the blade. This fraction will range from 1.0 when the blade is outside the beam to zero when it's completely blocking the beam. How far you'll have to move the blade depends on the size of the beam, and you'll measure beams that are as large as 1 cm and as small as 0.01 cm (perhaps even less). Thus the blade movement has to be very precise and controllable.

2 Theory

The strength of the electric field of the simplest mode of a laser is radially symmetric and given by the Gaussian profile $\mathcal{E}(r) = \mathcal{E}_0 e^{-(r/w)^2}$ where r is the radial distance from the beam axis and w is the characteristic beam radius. Note that there is no sharp boundary of the beam. Since the intensity I is proportional to $\mathcal{E}^2$, the radial dependence of the intensity is given by $I(r) = I_0 e^{-2(r/w)^2}$. Such a Gaussian beam profile is illustrated in Fig. 1, where the distance from the beam axis is measured in units of the beam radius.

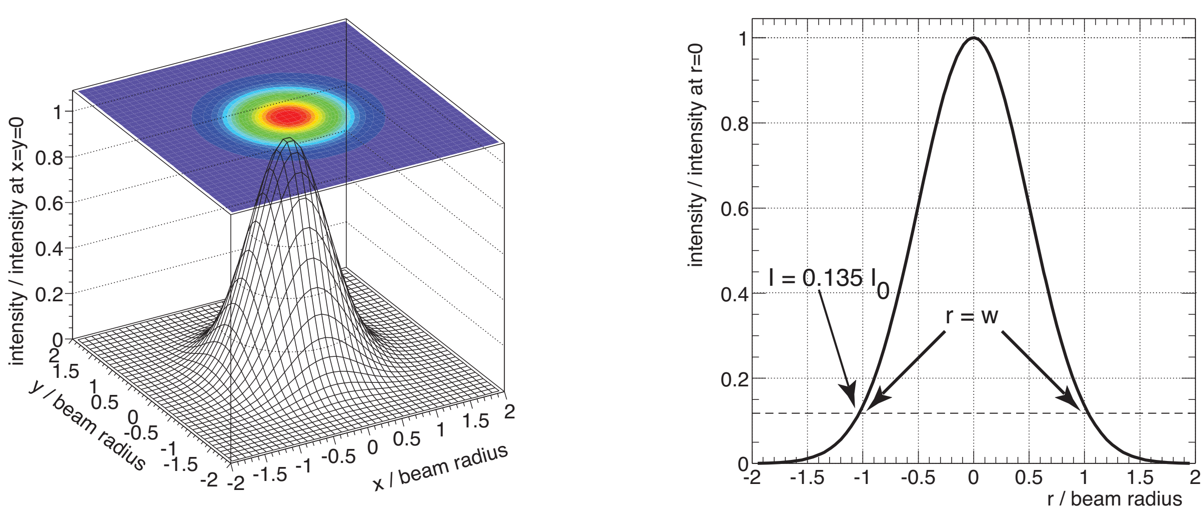

Figure 1: Gaussian beam profile in the x-y plane perpendicular to the beam axis (left side) and a projection of the profile on the radial distance $r = \sqrt{x^2 + y^2}$ from the beam axis (right side). The beam radius depends on the position along the beam axis if it is focussed or diverging.

As the laser beam propagates its diameter may change: the beam may either converge or diverge. The minimal size spot, where the beam has a radius w_0, is called the beam waist (see Fig. 2). In general, the beam radius is $w(z) = w_0\sqrt{1 + (z/z_0)^2}$ where $z_0 \equiv \pi w_0^2/\lambda$ and is called the Rayleigh

length. In the far field, *i.e.* at a distance $z \gg z_0$ from the minimum at $z = 0$, the beam radius is given by $w(z) = \lambda z/(\pi w_0)$. A measurement of the beam radius as function of z therefore allows the determination of w_0 for known wavelength or, if the waist can be measured, determination of the wavelength.

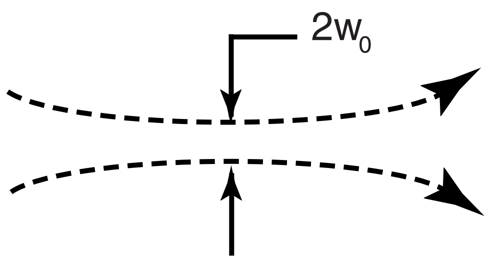

Figure 2: Schematic illustration of the profile of a Gaussian beam near a focus.

3 Setup

Measuring the transverse shape of the beam from the Helium-Neon laser (HeNe) is too difficult to start with - you should do that last. To start, set up your beam line with three lenses as shown in Fig. 3. The purpose of the first lens is to diverge the beam so that the second lens can form a larger beam that's easier to measure. Thus it could even be a diverging lens, but its shown as converging in Fig. 3. The second lens focusses the beam to a small spot, and you'll measure the beam's profile at several places along its length, starting from near this lens. The third lens is simply an auxiliary, because the laser beam itself may be too large to fit into the $\sim$7 mm hole in the photometer where the light sensor is. So this lens should be arranged to refocus the beam onto the detector's light sensor. You should arrange it so the beam is not focussed to a tiny spot there, but a disk a few mm in diameter.

Once you have focussed the light onto the detector you may find that the intensity of the laser light is larger than the range of your photometer allows for. But you know from a previous lab about a simple way to reduce the light intensity. Using the two polarizers from your optics kit you can first polarize the laser light linearly (right after the beam leaves the laser), and then adjust the polarization axis of the second polarizer, mounted just behind the first one, such that the laser light intensity passing the polarizer pair is just below the maximum range of the photometer. Use of this scheme may result in periodic drift if the laser has not had a long enough warmup time. Be sure to watch for such behavior.

You need to measure the intensity profile of this beam near the focus to determine the waist, and far from the focus to determine the divergence angle. You should therefore make about 10 or more cuts across the beam at various distances from the focusing lens (see Fig. 3), and each cut should comprise at least 20 points, so there is a lot of data to take. Moreover, after you finish, you will need to either move the lenses to change something, or swap the lenses, and then repeat the measurements.

The heart of the experiment is the blade motion. To accomplish this we have disassembled the Michelson interferometer bases, turned them over, and will use their precise micrometer motion control to move the blade. First you need to mount the blade on the aluminum arm with the tiny magnets as shown in Fig. 4 (use two above to hold the blade and one below to anchor the whole assembly). Be *exquisitely* careful with these magnets - they're easily lost and not easily replaced.

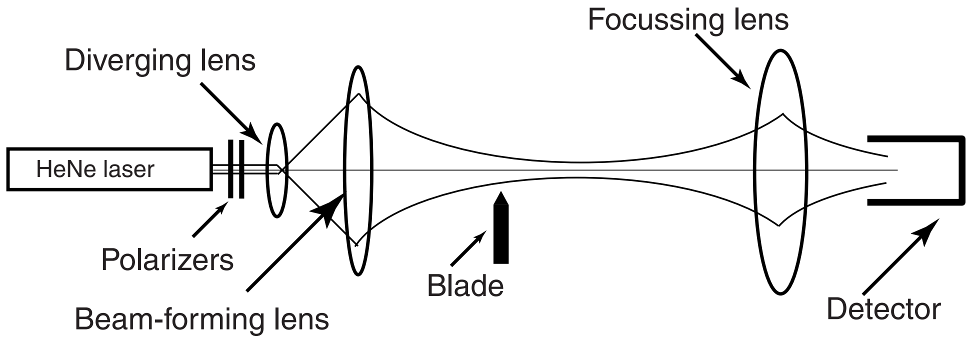

Figure 3: This shows the overall optical setup. The HeNe should be mounted on the table, independent of the other components. The first two lenses should be on the optical bench, near the end far from the HeNe. The inverted Michelson base and blade should be separately sitting on the table beyond the end of the bench. The focusing lens and detector should be independently mounted beyond the end of the Michelson base.

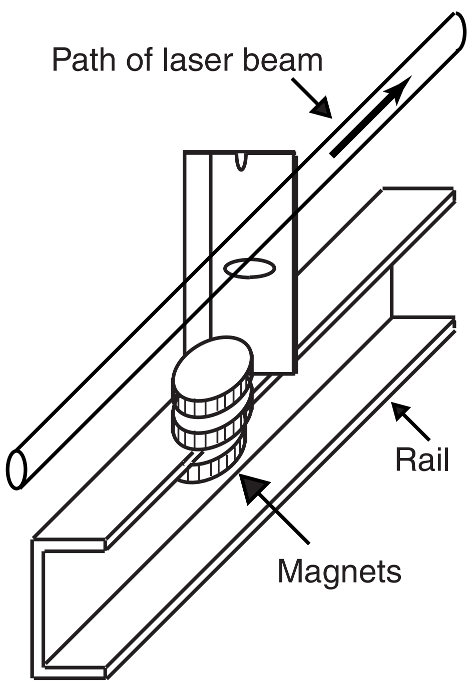

Figure 4: Schematic of blade assembly.

The laser beam should be arranged to travel parallel to the rail, just outside of the sharp edge of the razor blade, so that when you turn the micrometer screw, the blade advances into the beam and eventually blocks it completely. Finally, the photometer should be moved into a place where the entire beam shines onto the detector region. It may be that the beam is larger than the detector aperture, and if so, you should use an auxiliary lens to focus it into the hole so that **all** of the light is detected (see Fig. 3).

In order to make these measurements in a place where the beam diameter is large, say few mm or 1 cm or so, the razor blade should be mounted on the beam just above the micrometer screw so that its motion is the same as the micrometer motion. Since the scale of the micrometer screw is on the

bottom, you'll need to place a mirror flat on the table to see it (don't forget - it will be backwards). However, as you come near the waist, the radius of the beam will be tiny, perhaps only 1/2 mm or so, and therefore the 20 data points will have to be **very** closely spaced. If you mount the razor blade closer to the pivot point of the rail, say 90% of the way to the pivot, there will be an effective lever arm of 10:1 so that the blade moves only 1/10 of the reading of the micrometer screw. This trick should enable very tiny movements indeed.

4 Measurements

You obtain your data by moving the blade in small increments into the beam such that gradually less and less of the laser light strikes the detector (see the left side of Fig. 5). You will be measuring the integral of the remaining beam profile, that is, all the light that passes the razor blade. A correct calculation of the total transmitted light energy would involve a two-dimensional integral. Although the integration in one dimension is straightforward, the other is intractable. No analytic form is known, and so it is tabulated as the "error function", often abbreviated "erf" (see the right side of Fig. 5).

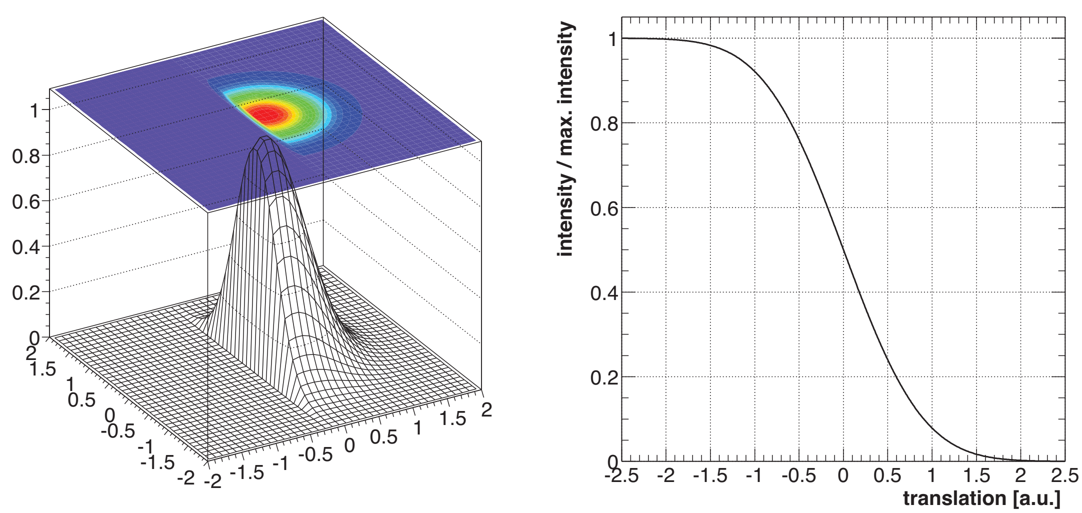

Figure 5: The left side shows how the Gaussian beam is cut by a razor blade that has advanced almost to its center. The right side schematically shows the error function, *i.e.* the intensity relative to the maximum intensity as function of the translation of the blade into the laser beam.

You can avoid this problem completely if you are careful to take equal size steps with the micrometer screw and then simply subtract adjacent data points. This will give a good approximation to the derivative and hence the beam profile, similar to the right hand side of Fig. 1. Then you can determine the width from the $1/e^2$ points of your curves where the intensity has dropped to 13.5% of the maximum value. Make sure that the beam spot on the photometer is not larger than the active area of the detector.

For a particular lens setup, repeat this procedure at various positions along the beam. You can achieve these positions in two different ways. One way is to move the blade along the aluminum rail,

but if you do this you have to account for how the lever arm affects the motion. The other way is to move the entire Michelson base. But you should **NOT** move the lenses because then you will change both the waist and z_0. You should be able to measure beam diameters from 1 cm down to less than 0.01 cm. Obviously, the smaller the beam spot the more difficult the measurement will be until, at some point, the precision of your apparatus will not be sufficient to resolve any smaller beam spots. Plot the beam diameter as function of the blade's distance from the "beam forming lens" of Fig. 3 and compare to the theoretical expectation.

One of the objectives of this experiment is to measure w_0 for a particular beam. The best way to do this is to differentiate the equation $w(z) = w_0\sqrt{1+(z/z_0)^2}$ to find the divergence angle $\theta = dw/dz = w_0 z/(z_0^2\sqrt{1+(z/z_0)^2}\ )$. For $z \gg z_0$ this becomes $dw/dz = w_0/z_0 = \lambda/\pi w_0$ (compare this to $\theta = 1.22\lambda/D$ for an aperture). If you plot your measured w *vs.* your measured z, its slope can be used to find w_0. Note that there is no "right answer" or value that you are "supposed" to get, because each beam is different. You will be judged on how well you perform the experiment, not on what answer you get.

Now that you have extracted the size w_0 of the beam waist from your measurements you can also extract the Rayleigh length z_0. Again, you need to analyze the z dependence of the beam's width, *i.e.* the divergence $\theta = dw/dz$ of the beam. In the far field, this is simply $dw/dz = w_0/z_0 = \lambda/\pi w_0$. Since you know w_0, you can determine z_0. What do you find? How does this compare with your expectation? Discuss the physical meaning of the Rayleigh length z_0.

To get a better feeling for the meaning and interplay between the two parameters w_0 and z_0, which are relevant for the propagation of Gaussian beams, change your setup to prepare a different beam. You can swap the diverging and beam-forming lenses (see Fig. 3) or replace one (or both) of them with another lens with different focal length. You can even change the distances between your lenses. It is up to you to decide how you want to change your setup. However, try to prepare a beam that is significantly different from the one you have studied already. Repeat the same measurements you did before for this second setup. If you want to prepare yourself for the last part of this lab experiment you should try to prepare a somewhat narrower beam than you had first. That way you can practice and improve your skills in measuring small beam diameters (you will need them).

For the last part of the experiment remove the diverging and beam-forming lenses. You will try to measure the waist and Rayleigh length of the plain HeNe beam directly. Be careful to check the size of the beam spot on the photometer. You might be able to also remove the auxiliary lens you have used to focus all the light on the active area of the photometer. If the beam spot is too large, keep that lens. Carefully repeat your measurements of w_0 and z_0 (it might be difficult!) for the HeNe beam and discuss your results. From your value of z_0 and the positions of the blade, locate the position of the beam waist (where is $z = 0$?).

This measurement may involve longer distances that the ones you had with beams focussed by the lenses. Since it may be awkward to transport your beam scanning setup to a bench across the room, try to think of a way to perform the needed measurements on your own table.