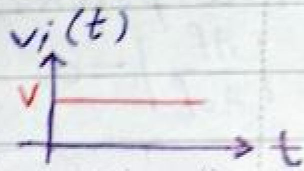
4.5 Step Response



$v_i(t) = V \rightarrow V_i(s) = V/S$

$$\frac{V_o(s)}{V_i(s)} = \frac{V_o(s)}{V/S} = \frac{A_of}{(S/\omega_o)^2 + 2K(S/\omega_o) + 1}$$

$$\therefore \frac{V_o(s)}{A_ofV} = \frac{1}{S\left[(S/\omega_o)^2 + 2K(S/\omega_o) + 1\right]}$$

$$\therefore S_{1,2f} = -K\omega_o \mp \omega_o\sqrt{K^2-1}$$

(a) if $K=1$ then the Two poles are coinside to critically damping case

$$\therefore \frac{V_o(t)}{VA_of} = 1 - (1+\omega_o t)e^{-\omega_o t}$$

(b) If $K<1$, the two poles are complex conjucate corresponding to underdamping case.

Where the response is a sinusoid with amplitude decayes with time.

$$S_{1,2f} = -\omega_o K \mp j\omega_o\sqrt{1-K^2}$$

$\omega d \Rightarrow$ damping freqy $= \omega_o\sqrt{1-K^2}$

$$\frac{v_o(t)}{VA_of} = 1 - \left(K \frac{w_o}{w_d} \sin w_d t + \cos w_d t\right) e^{-Kw_o t}$$

oscillation

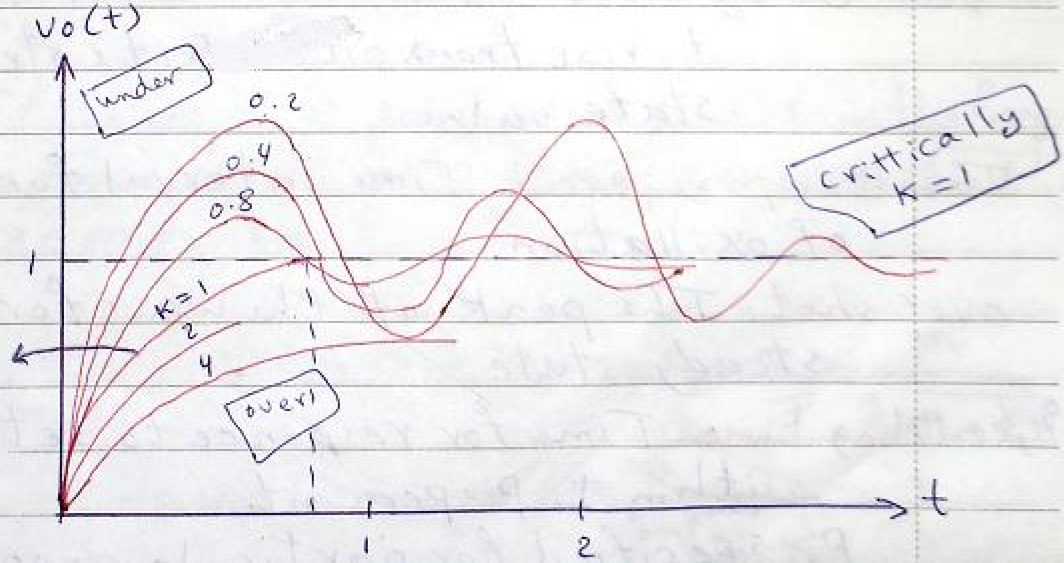
(c) If $K > 1$, Both poles are real and (-ve) Which corresponding to overdamped case Where the responce approaches its final steady state value after a time without oscillation.

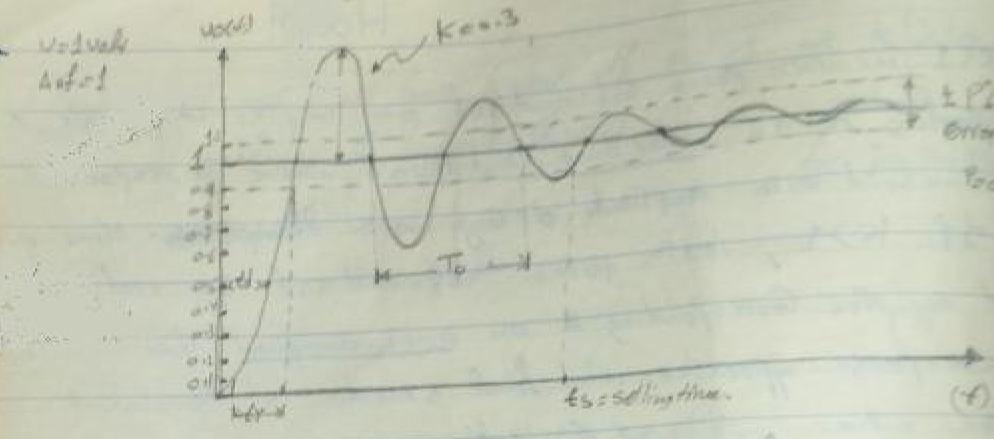
$$\frac{v_o(t)}{VA_of} = 1 - \frac{1}{2\sqrt{K^2-1}} \left(\frac{1}{K_1} e^{-K_1 w_o t} - \frac{1}{K_2} e^{-K_2 w_o t}\right)$$

Where $s_1f = -w_o K_2$, $s_2 f = -w_o K_1$

$K_1 = K - \sqrt{K^2-1}$, $K_2 = K + \sqrt{K^2-1}$

if $4K^2 \gg 1$ then $v_o(t)/VA_of = 1 - e^{-w_o t/2K}$





Step Response of Two pole F.B amp. for a damping factor K=0.3

tr: Rise time : time for the wave form to rise from 0.1 to 0.9 of it's study state value.

td: Time for the waveform to rise from 0 to 0.5 of it's study state value.

Damp Period (To): Time interval for 1 sycle of osillations

Over shot : Peak excursion above the steady state value.

Settling time : Time for response to settle to within ± P Persent.

P : specified for a particular application.

Unit Step Response.

$y = vo(t) / v\,Aof$

$x = t/To$, $To = 2\pi/\omega o$ under damp Period.

The max. of y are obtained at $x = xm$

$xm = \omega o\, tm / 2\pi = m / 2(1-K^2)^{1/2}$ —— ①

$ym = \frac{vo(tm)}{V Aof} = 1 - (-1)^m e^{-\pi K m}$ —— ②

$\text{overshoot} = e^{-2\pi K xm}$

where m is integer .

* $y = v_o(t)/VA_of$ take $[x = t/T_o]$ & $[T_o = 2\pi/\omega_o]$

→ underdamping period

* The max of y → $x = x_{max}$ → $x_m = \omega_o t_m / 2\pi$

$$\therefore X_m = \frac{m}{2\sqrt{1-k^2}}$$

$$y_m = 1 + (-1)^m e^{-2\pi K x_m}$$

$$\text{overshot} = e^{-2\pi K x_m}$$

* The max. occure for all odd value of m.
* The min. occure for all even value of m

Ex. ⓐ If $K = 0.5$, Calculate the percent max. overshot in the step responce for two pole F.B. amp.

ⓑ If there is 10 percent overshot in the freqy responce, what is the percent overshot in the step responce.

Ans. (a) The max. occure at $m=1$

$y = x_1 = 1/(2\sqrt{1-k^2}) = 0.5773$

$$\text{max overshot} = e^{-2\pi k x_1} = 0.163 = 16.3\%$$

(b) In freqy responce, the max overshot $= \dfrac{1}{2k\sqrt{1-k^2}} - 1 = 0.1$ (y)

$$k^2(1-k^2) = 0.2066$$

$$\therefore k^2 - k^4 - 0.2066 = 0$$

$$\therefore k^2 = 1/2 \mp \frac{\sqrt{1-4 * 0.2066}}{2} = 0.5 \pm 0.208$$

(1) $k^2 = 0.708 \Rightarrow k = 0.8416$

$\therefore x_1 = 1/2\sqrt{1-k^2} = 0.925$

max overshot $= e^{-2\pi k x_1} = \boxed{0.747\%}$

(2) $k^2 = 0.292 \Rightarrow k = 0.54 \Rightarrow x_1 = 0.594$

$\therefore$ max overshot $= e^{-2\pi k x_1} = \boxed{13.32\%}$ لأكبر

$\therefore$ The max. overshot in the step preresponse corresponding to $k = 0.54$ is 13.32%

Ex2/ Given Two Pole amp. with $\omega_1 = 1$ Mrad/sec, $\omega_2 = 0.1$ Mrad/sec

a) Find the max value of the loop gain for which the step response

b) At What time will the Peak occure.

c) calculate the magnitude of the first min. of the step response at {the time at which occure}

d) Verify That for $K = 0.707$, the max. overshoot is 4.3 Percent.

Ans:- a) The max. occure at $m=1$ & overshoot $= e^{-\pi k m/\sqrt{1-k^2}}$, So

$0.1 = e^{-\pi k/\sqrt{1-k^2}} \Rightarrow 2.3 = \frac{\pi k}{\sqrt{1-k^2}} \Rightarrow 0.537 = \frac{k^2}{1-k^2} \Rightarrow$

$\Rightarrow k^2 = 0.349 \rightarrow \therefore K = 0.591$

$Q = 1/2K = 0.845$, $W_0 = Q(W_1 + W_2) = 1.014$ M rad/sec.

$W_0 = \sqrt{W_1 W_2 (1 + \beta A_0)} \Rightarrow \beta A_0 = \frac{W_0^2}{W_1 W_2} - 1 = 4.14$

b) $\frac{t_m w_0}{2\pi} = \frac{m}{2\sqrt{1-k^2}}$ & The Peak occure at $m=1$

$t_{peak} = \frac{\pi}{w_0\sqrt{1-k^2}} = \frac{\pi}{1.014 \times 10^6 \sqrt{1-0.591^2}} = 3.84$ M sec.

c) $Y_2 = 1 - e^{-2\pi k x_2}$, first min occure at $m=2$

$x_2 = \frac{2}{2\sqrt{1-k^2}} = 1.24$ $\therefore y_2 = 0.98999$

$\frac{t_2 w_0}{2\pi} = \frac{2}{2\sqrt{1-k^2}} \Rightarrow t_{2\,peak} = \frac{2\pi}{w_0\sqrt{1-k^2}} = 7.68$ M sec

d) $m=1$, $x_1 = 1/2\sqrt{1-k^2} = 1/2\sqrt{1-0.707^2} = 0.707$

over Shoot $= e^{-2\pi k x_1} = 0.043 = 4.3\%$