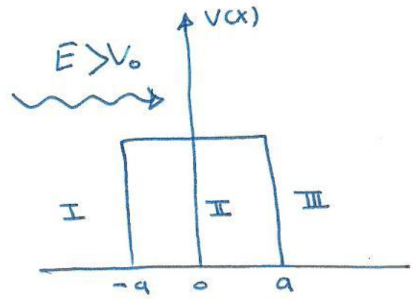
Potential barrier :

$V(x) = 0 \qquad x < -a$

$V(x) = V_0 \qquad -a < x < a$

$V(x) = 0 \qquad x > a$



(i) $E > V_0$

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_1 = E\psi_1 \implies \frac{d^2\psi}{dx^2} + k^2\psi_1 = 0$$ (I) في المنطقة

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi_2 + V_0\psi_2 = E\psi_2 \implies \frac{d^2\psi_2}{dx^2} + k'^2\psi_2 = 0$$ (II) في المنطقة

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_3}{dx^2} = E\psi_3 \implies \frac{d^2\psi_3}{dx^2} + k^2\psi_3 = 0$$ (III) في المنطقة

where $k = \sqrt{\frac{2mE}{\hbar^2}}$, $k' = \sqrt{\frac{2m}{\hbar^2}(E - V_0)}$

Solution

$$\psi_1 = A e^{ikx} + B e^{-ikx}$$

$$\psi_2 = C e^{ik'x} + D e^{-ik'x} \qquad D \neq 0$$

$$\psi_3 = F e^{ikx}$$

Boundary Condition (B.C) الشروط الحدودية

at $x=-a$, $\psi_1(x)=\psi_2(x)\Big|_{x=-a}$

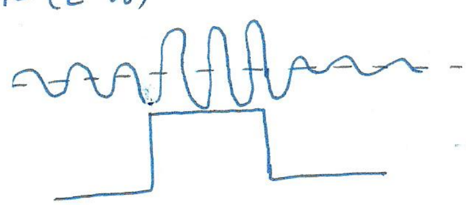
and $\dfrac{d\psi_1(x)}{dx}=\dfrac{d\psi_2(x)}{dx}\Big|_{x=-a}$

at $x=a$, $\psi_2(x)=\psi_3(x)\Big|_{x=a}$

$\dfrac{d\psi_2(x)}{dx}=\dfrac{d\psi_3(x)}{dx}\Big|_{x=a}$

$$T=\frac{1}{1+\frac{V_0^2}{4E(E-V_0)}\sinh^2(2k'a)} \quad , \quad R=1-T$$

$T+R=1$

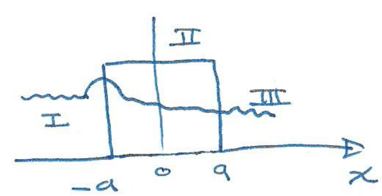


ii) $E<V_0$

$\psi_1(x)=Ae^{ikx}+Be^{-ikx}$

$\psi_2(x)=Ce^{-\alpha x}+De^{\alpha x}$

$\psi_3(x)=Fe^{ikx}$



where $\alpha=\sqrt{\frac{2m}{\hbar^2}(V_0-E)}$

* by using Boundary Condition (B.C), we can find the constants A, B, C, D, F.

* by using probability current density, we can find

$$T = \frac{1}{1 + \frac{V_0^2}{4E(V_0 - E)} \sinh^2(2\alpha a)}$$

This is called Tunnel effect. For example, alpha particle emission from heavy nuclei explained as a tunneling process through the nuclear barrier.

وهذا التأثير يعني عبور الجسيم حاجز الجهد بالرغم
من كون $(E<V_0)$ وهذا غير مسموح به كلاسيكياً لكن كمياً يمكن حدوثه
وهو ما يسمى "التأثير النفقي"