ii) Gauss_Jordon Elimination Method

In this method the elements above diagonal are made zeros at the same time that zeros are greaten below diagonal. Usually get on diagonal matrix, that is transform the coefficients matrix into diagonal matrix, when this has been accomplished the variables can be computed directly without using backward substitution, so we get on;

$$\begin{bmatrix} a''_{11} & 0 & 0 \\ 0 & a'_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b''_1 \\ b'_2 \\ b_3 \end{bmatrix}$$

So ; $x_1 = \frac{b''_1}{a''_{11}}$, $x_2 = \frac{b'_2}{a'_{22}}$ and $x_3 = \frac{b_3}{a_{33}}$. If matrix for $n \times n$ dimension $x_n = \frac{b_n}{a_{nn}}$.

this method can be completed for the Gauss_Elimination Method.

Example: Solve the equations system in previous example by Gauss_Jordon Method.

Solution: From previous example by Gauss_Elimination Method a form ;

$$\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -1 \\ 8 \end{bmatrix}$$

calculate d_1 as $d_1 = \frac{coeff.of\ x_3\ in\ eq.(4)}{coeff.of\ x_3\ in\ eq.(6)} = \frac{a_{23}}{a_{33}} = \frac{-1}{4}$

and ; $eq.(4) - d_1\, eq.(6)$,so $\begin{bmatrix} 2 & 3 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ 8 \end{bmatrix}$

then; $x_2 = 1$ ....(٧)

calculate d_2 as $d_2 = \frac{coeff.of\ x_3\ in\ eq.(1)}{coeff.of\ x_3\ in\ eq.(6)} = \frac{a_{13}}{a_{33}} = \frac{1}{4}$

and ; $eq.(1) - d_2\, eq.(6)$,so $\begin{bmatrix} 2 & 3 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 8 \end{bmatrix}$

$2x_1 + 3x_2 = 5$ ....(٨)

calculate d_3 as $d_3 = \frac{coeff.of\ x_2\ in\ eq.(8)}{coeff.of\ x_2\ in\ eq.(7)} = \frac{a_{12}}{a_{22}} = 3$

and ; $eq.(8) - d_2\, eq.(7)$,so $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 8 \end{bmatrix}$

$x_1 = 2$ ....(٩)

now from eq.(٩) $x_1 = 1$,and from eq.(٧) $x_2 = 1$ and ,from eq.(٦) $x_3 = 2$

then $X = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

Exercises:

١. Solve previous exercises using Gauss_Jordon Method and show that the solution must be equal.

٢. Find solution vector for system of equations, using Gauss_Jordon Method:

$$x_1 + x_2 + x_3 = 3$$
$$5x_1 + x_2 - 2x_3 = 5$$
$$x_1 + 2x_2 + x_3 = 4$$

The Norm:

In Iteration Methods (Indirect Methods), we need to calculate absolute and relative error for variables vector at each iteration using Norm principle for vectors, that in three types as;

- First_Order Norm(L_1)
- Second_Order Norm(L_2)
- Infinity Norm(L_∞)

First Order Norm(L_1):The sum of absolute values for vector's elements, which denoted by $L_1(X)$ or $\|X\|_1$ for vector X as;

$$\|X\|_1 = \sum_{i=1}^{n} |x_i| = |x_1| + |x_2| + ... + |x_n|$$

for example for $X = \begin{bmatrix} 1 \\ -1 \\ -5 \\ 3 \end{bmatrix}$ then $\|X\|_1 = |1| + |-1| + |-5| + |3| = 10$

and $Y = \begin{bmatrix} 3 \\ 2 \\ 1 \\ 7 \end{bmatrix}$ then $\|X - Y\|_1 = \sum_{i=1}^{n} |x_i - y_i| = |x_1 - y_1| + |x_2 - y_2| + ... + |x_n - y_n|$

$$= |1-3| + |-1-2| + |-5-1| + |3-7| = 15$$