القيمة المطلقة (Absolute Value) أو المعيار (Modules)

القيمة المطلقة للعدد المعقد $Z = x + iy$ ويرمز لها بالرمز $|Z|$ هي

$|Z| = \sqrt{x^2 + y^2}$ وهي عدد غير سالب

مثال

جد القيمة المطلقة للعدد $Z = 3 + 4i$ هي $|Z| = \sqrt{3^2+4^2} = 5$

بعض خواص القيم المطلقة(المعيار)

1) $|Z| = \sqrt{Z\overline{Z}} \Rightarrow |Z|^2 = Z\overline{Z}$

2) $|Z| = |\overline{Z}|$

3) $|Z_1 - Z_2| = |Z_2 - Z_1|$

4) $|Z_1 . Z_2| = |Z_1|\,|Z_2|$

5) $|Z_1.Z_2|^2 = |Z_1|^2.|Z_2|^2$

6) let $\lambda \in \Re$ then $|\lambda Z| = |\lambda|.|Z|$

7) $\operatorname{Im}(Z) \le |\operatorname{Im}(Z)| \le |Z|$

8) $\operatorname{Re}(Z) \le |\operatorname{Re}(Z)| \le |Z|$

9) $|Z|^2 = (\operatorname{Re}(Z))^2 + (\operatorname{Im}(Z))^2$

10) $|Z_1 + Z_2| \le |Z_1| + |Z_2|$

11) $\left|\frac{1}{Z}\right| = \frac{1}{|Z|}, Z \ne 0$

والآن سوف نحاول برهان بعض من هذه الخواص:

Proof (2): suppose z = x +iy and that $\bar{z} = x - iy$

$$|\overline{Z}| = |x - iy| = \sqrt{x^2 + (-y)^2} = \sqrt{x^2 + y^2} = |Z|$$

Proof (4):

$$|Z_1 . Z_2| = |(x_1+iy_1).(x_2+iy_2)| = |(x_1 x_2 - y_1 y_2 + i(y_1 x_2 + x_1 y_2)|$$

$$= ((x_1 x_2 - y_1 y_2)^2 + (y_1 x_2 + x_1 y_2)^2)^{1/2}$$

$$= \sqrt{(x_1^2 x_2^2 - 2x_1 x_2 y_1 y_2 + y_1^2 y_2^2 + y_1^2 x_2^2 + 2x_1 x_2 y_1 y_2 + y_2^2 x_1^2)}$$

$$= \sqrt{(x_1^2 x_2^2 + y_1^2 y_2^2 + y_1^2 x_2^2 + y_2^2 x_1^2)}$$

$$= \sqrt{(x_1^2 + y_1^2).(x_2^2 + y_2^2)} = \sqrt{(x_1^2 + y_1^2)} . \sqrt{(x_2^2 + y_2^2)}$$

$$= |Z_1| . |Z_2|$$

Proof (6):

$$|\lambda Z| = |\lambda(x + iy)| = |\lambda x + i\lambda y| = \sqrt{(\lambda x)^2 + (\lambda y)^2}$$

$$= \sqrt{\lambda^2 x^2 + \lambda^2 y^2}$$

$$= \lambda \sqrt{x^2 + y^2} = \lambda |Z|$$

Proof (8):

Suppose Re(Z) = x

If $x \geq 0 \Rightarrow Re(Z) = x = |Re(Z)|$

If $x < 0 \Rightarrow Re(Z) = -x$

$\therefore Re(Z) \leq |Re(Z)|$ …(1)

$|Re(Z)| = x = \sqrt{x^2}$

$\leq \sqrt{x^2 + y^2} = |Z|$

$\therefore |\mathrm{Re}(Z)| \leq |Z|$ ...(2)

from (1) & (2) we obtain $\mathrm{Re}(Z) \leq |\mathrm{Re}(Z)| \leq |Z|$

Proof (10): $|Z_1 + Z_2| \leq |Z_1| + |Z_2|$

By (1) we get $|Z_1 + Z_2|^2 = (Z_1 + Z_2)\overline{(Z_1 + Z_2)}$

$$= (Z_1 + Z_2)(\overline{Z}_1 + \overline{Z}_2)$$

$$= (Z_1\overline{Z}_1 + Z_1\overline{Z}_2 + \overline{Z}_1 Z_2 + Z_2\overline{Z}_2)$$

$$= |Z_1|^2 + Z_1\overline{Z}_2 + \overline{Z}_1 Z_2 + |Z_2|^2$$

Let $u = Z_1\overline{Z}_2$ & $\overline{u} = \overline{Z}_1 Z_2$

$$\therefore \quad Z_1\overline{Z}_2 + \overline{Z}_1 Z_2 = u + \overline{u} = 2\,\mathrm{Re}(u)$$

$$\leq 2|u| = 2|Z_1\overline{Z}_2|$$

$$= 2|Z_1||\overline{Z}_2|$$

$$= 2|Z_1||Z_2|$$

Since $|Z_1 + Z_2|^2 = |Z_1|^2 + Z_1\overline{Z}_2 + \overline{Z}_1 Z_2 + |Z_2|^2$

$$\leq |Z_1|^2 + 2|Z_1||Z_2| + |Z_2|^2$$

$$\leq (|Z_1| + |Z_2|)^2$$

By take the root

$\therefore |Z_1 + Z_2| \leq |Z_1| + |Z_2|$