

Subcritical Crack Growth

Subcritical crack growth (SCG) refers to the slow growth of a Subcritical flaw as a result of its exposure to the combined effect of stress and a corrosive environment. The combination of a reactive atmosphere and a stress concentration can greatly enhance the rate of crack propagation. For instance, silica will dissolve in water at a rate of 10^{-17} m/s, whereas the application of stress can cause cracks to grow at speeds greater than 10^{-3} m/s. The insidiousness of, and hence the importance of understanding, this phenomenon lies in the fact that as the crack tip advances, the material is effectively weakened and eventually can give way suddenly and catastrophically after years of service.

Experimental Details: Measuring Subcritical Crack Growth

The techniques and test geometries that have been used to measure subcritical crack growth in ceramics are several, but they share a common principle, namely, the subsection of a well-defined crack to a well-defined stress intensity K_I , and a measurement of its velocity v . The technique considered here, the advantages of which are elaborated upon below, is the double torsion geometry shown in Fig. 1. For the double torsion specimen, K_I is given by

$$K_I = PW_m \sqrt{\frac{3(1 + \nu)}{Wd^3d_n}} \dots\dots\dots(1)$$

where P is the applied load and ν is Poisson's ratio; all the other symbols and dimensions are defined in Fig. 1. The measurements are carried out as follows:

1. A starter crack is introduced in a specimen, and a load is applied, as shown in Fig. 1. As a result, the starter crack will grow with time.
2. The rate of crack growth is measured, usually optically. For instance, two marks are placed on the specimen surface, the time required for the crack to propagate that distance is measured. The crack velocity is then simply $v = \Delta c / \Delta t$. The major advantage of this geometry is embedded in Eq. 1. Since K_I is not a function of crack length, it follows that v is also not a function of crack length; i.e., a constant crack velocity should be observed for any given load, which greatly simplifies the measurement and analysis. The disadvantage of the technique, however, is that it requires reasonably large samples that require some machining.
3. The experiment is repeated under different loading conditions, either on the same specimen, if it is long enough, or on different specimens, if not.

If explored over a wide enough spectrum, a **In v** versus **K_I** plot will exhibit four regions, as shown in Fig. 2a:

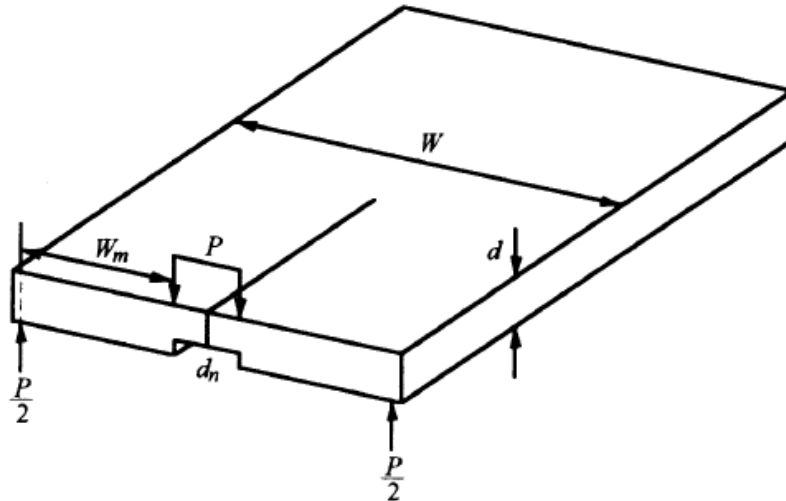


Figure 1. Schematic of double torsion specimen

- A threshold region below which no crack growth is observed
- Region I, where the crack growth is extremely sensitive to K_I and is related to it by an exponential function of the form

$$v = A^* \exp \alpha K_I$$

where A^* and α are empirical fitting parameters

- Region II, where the crack velocity appears to be independent of K_I
 - Region III, where the crack growth rate increases even more rapidly with K_I than in region I
- To measure n (defined in Eq. (3)) directly from velocity versus K_I curves is often difficult and time consuming. Fortunately, a simpler and faster technique, referred to as a dynamic fatigue test (not to be confused) with the normal fatigue test discussed below), is available. In this method, the strain rate dependence of the average strength to failure is measured; i.e., the samples are loaded at varying rates and their strength at failure is recorded. It can be shown that the mean failure stress σ_1 at a constant strain rate $\dot{\epsilon}_1$ is related to the mean failure stress σ_2 at a different strain rate $\dot{\epsilon}_2$ by:

$$\left(\frac{\sigma_1}{\sigma_2} \right)^{n+1} = \frac{\dot{\epsilon}_1}{\dot{\epsilon}_2} \dots\dots\dots(2)$$

Hence by measuring the stress to failure at different strain rates, n can be directly calculated from this relationship.

Typical v versus K data for soda-lime silicate glasses tested as a function of humidity are shown in Fig. 2b, where the following salient points are noteworthy:

- The rate of crack propagation is a strong function of the applied stress.

- Humidity has a dramatic effect on v ; increasing the relative humidity in the ambient atmosphere from 0.02 to 100 percent, results in a greater than 3 orders-of-magnitude increase in v .
- Clear identification of the three regions just described is possible.
- The presence of a threshold K_I is not clearly defined because of the difficulty of measuring crack velocities that are much smaller than 10^{-7} m/s.
- The dramatic effect of increasing the temperature on crack velocity is shown in Fig. 2c. The velocity increases by about 2 orders of magnitude over a temperature range of $\approx 100^\circ\text{C}$, typical of thermally activated processes.

Lifetime Predictions during SCG

from a design point of view, the central question is, How long can a part serve its purpose reliably? The conservative approach, of course, would be to design with stresses that are below the thresholds. An alternative approach is to design a part to last for a certain lifetime, after which it would be replaced or at least examined for damage

$v = A'' K_I^n$ (3) & $t_f = (\text{const}) e^{-\beta K_I / (RT)}$ are used to predict life time for (SCG)

Ex:

For silica glass tested in ambient temperature water, $v_0 = 3 \times 10^{-22}$ m/s and $\beta = 0.182 \text{ m}^{5/2}$. Estimate the effect of increasing K_I from 0.4 to 0.5 MPa · m^{1/2} on the lifetime.

Answer

By noting that $K_I = \Psi \sigma_a \sqrt{\pi c}$, Eq. (12.39) can be rewritten as

$$t_f = (\text{const}) e^{-\beta K_I / (RT)}$$

Here the dependence of the preexponential term on K_I was ignored, which is a good approximation relative to the exponential dependence. Substituting the appropriate values for K_I and β in this expression, one obtains

$$\frac{t_{f@0.4}}{t_{f@0.5}} = \frac{\exp\left(-\frac{0.182 \times 0.4 \times 10^6}{8.314 \times 300}\right)}{\exp\left(-\frac{0.182 \times 0.5 \times 10^6}{8.314 \times 300}\right)} = 1476$$

In other words, a decrease in K_I from 0.5 to 0.4 MPa · m^{1/2} increases the lifetime by a factor of about 1500!

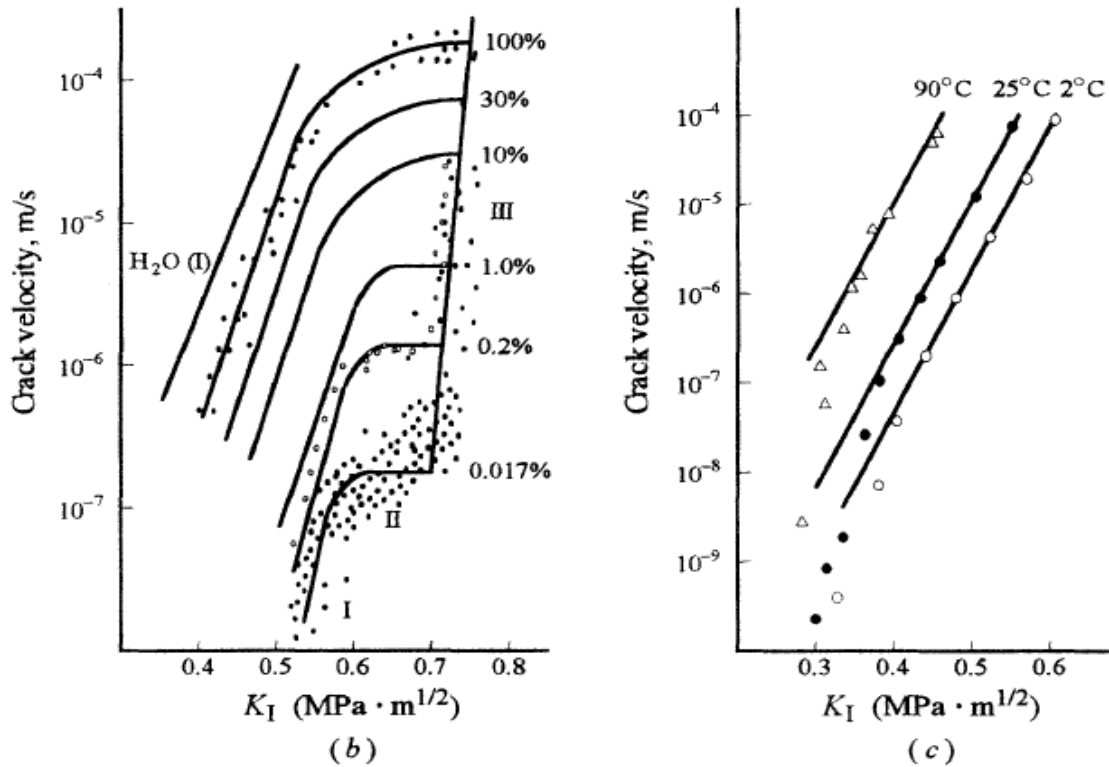
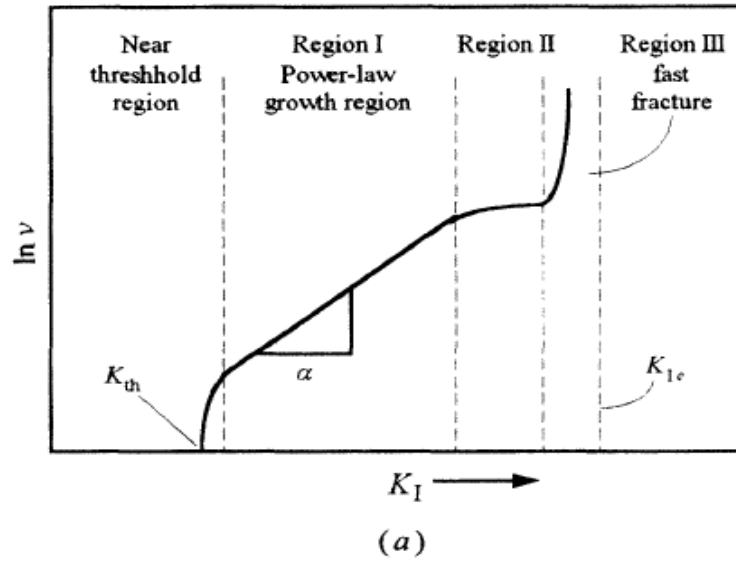


Figure 2. (a) Schematic of crack propagation rates as a function of K_I , where four stages are identified. (b) Actual data for soda-lime glass tested in N_2 gas of varying relative humidity shown on the right-hand side (c) Temperature dependence of crack propagation in same glass in water.

H.W.

Typical crack growth data for a glass placed in a humid environment are listed below. Calculate the values of A'' , A , and n (see Eq. (12.30)), given that $K_{Ic} = 0.7 \text{ MPa} \cdot \text{m}^{1/2}$. What are the units of A'' ?

Stress intensity, $\text{MPa} \cdot \text{m}^{1/2}$	0.4	0.5	0.55	0.6
Crack velocity, m/s	1×10^{-6}	1×10^{-4}	1×10^{-3}	1×10^{-2}

Answer: $n = 22.5$, $A'' = 772$, $A \approx 0.25 \text{ m/s}$