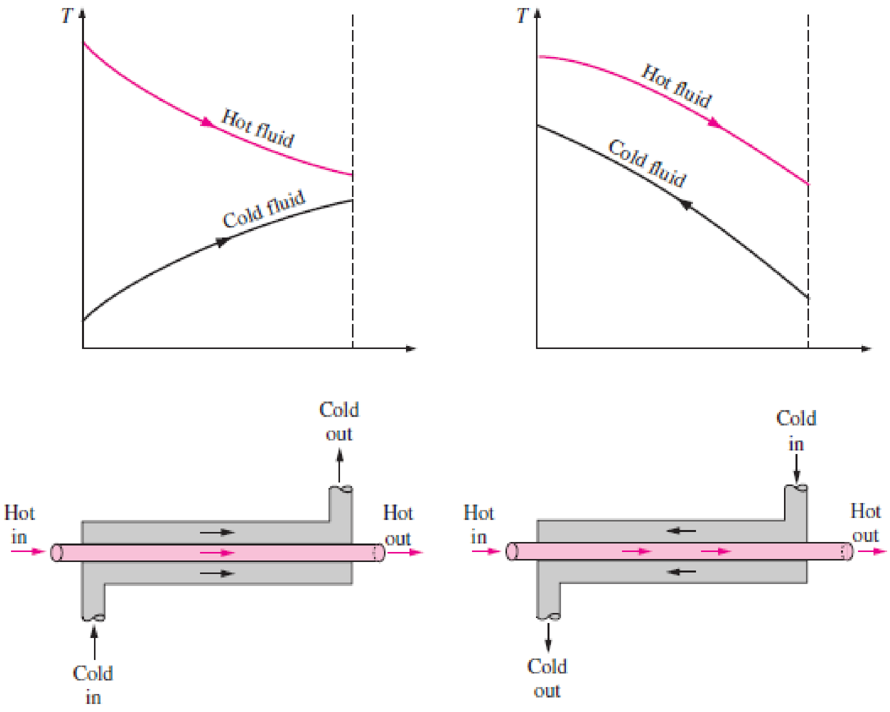
HEAT EXCHANGERS

Heat exchangers are devices that facilitate the *exchange of heat* between *two fluids* that are at different temperatures while keeping them from mixing with each other. Heat exchangers are commonly used in practice in a wide range of applications, from heating and air-conditioning systems in a household, to chemical processing and power production in large plants. Heat exchangers differ from mixing chambers in that they do not allow the two fluids involved to mix. In a car radiator, for example, heat is transferred from the hot water flowing through the radiator tubes to the air flowing through the closely spaced thin plates outside attached to the tubes.

TYPES OF HEAT EXCHANGERS

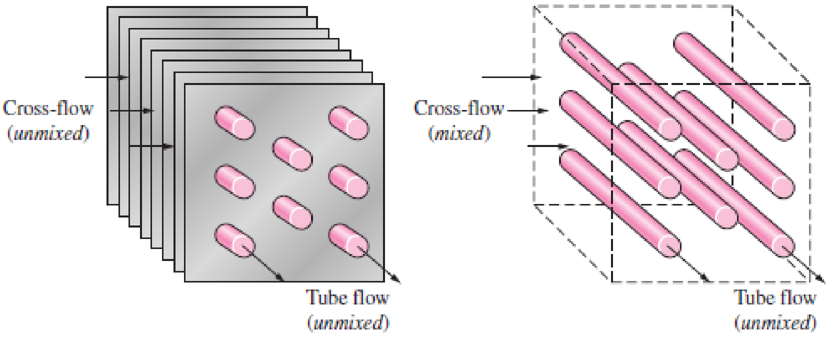
The simplest type of heat exchanger consists of two concentric pipes of different diameters, as shown in Figure 13–1, called the **double-pipe** heat exchanger. One fluid in a double-pipe heat exchanger flows through the smaller pipe while the other fluid flows through the annular space between the two pipes. Two types of flow arrangement are possible in a double-pipe heat exchanger: in **parallel flow,** both the hot and cold fluids enter the heat exchanger at the same end and move in the *same* direction. In **counter flow,** on the other hand, the hot and cold fluids enter the heat exchanger at opposite ends and flow in *opposite* directions. Another type of heat exchanger, which is specifically designed to realize a large heat transfer surface area per unit volume, is the **compact** heat exchanger. The ratio of the heat transfer surface area of a heat exchanger to its volume is called the *area density* β. A heat exchanger with $\beta = 700\ m^2/m^3$ (or 200 ft2/ft3) is classified as being compact. Examples of compact heat exchangers are car radiators ($\beta = 1000\ m^2/m^3$).



(*a*) Parallel flow

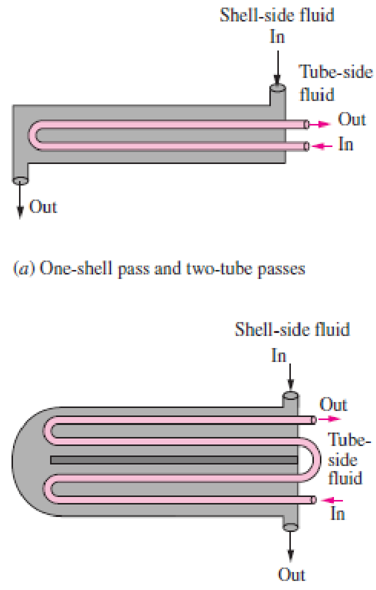
(*b*) Counter flow

In compact heat exchangers, the two fluids usually move *perpendicular* to each other, and such flow configuration is called **cross-flow.** The cross-flow is further classified as *unmixed* and *mixed flow,* depending on the flow configuration, . In (*a*) the cross-flow is said to be *unmixed* since the plate fins force the fluid to flow through a particular inter fin spacing and prevent it from moving in the transverse direction (i.e., parallel to the tubes). The cross-flow in (*b*) is said to be *mixed* since the fluid now is free to move in the transverse direction. Both fluids are unmixed in a car radiator. The presence of mixing in the fluid can have a significant effect on the heat transfer characteristics of the heat exchanger.



(a) Both fluids unmixed

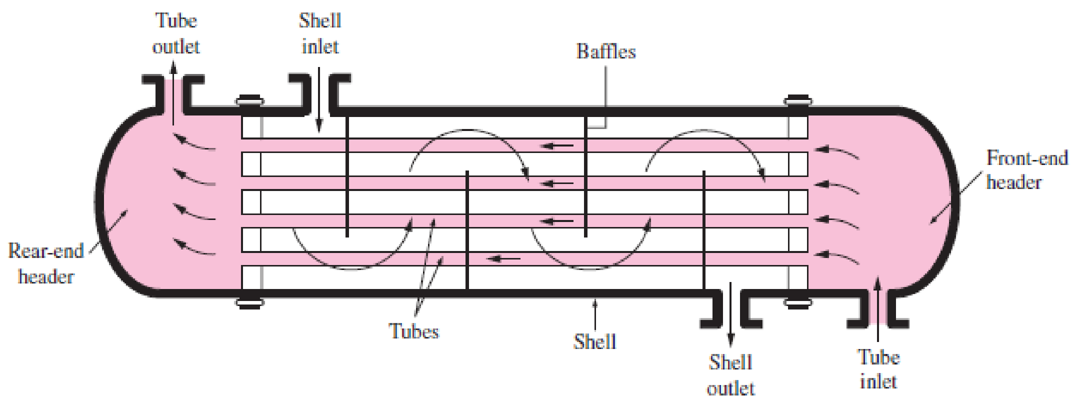
(b) One fluid mixed, one fluid unmixed



(a) One-shell pass and two-tube passes

(b) Two-shell passes and four-tube passes

Perhaps the most common type of heat exchanger in industrial applications is the **shell-and-tube** heat exchanger, shown in Figure. Shell-and-tube heat exchangers contain a large number of tubes (sometimes several hundred) packed in a shell with their axes parallel to that of the shell. Heat transfer takes place as one fluid flows inside the tubes while the other fluid flows outside the tubes through the shell. *Baffles* are commonly placed in the shell to force the shell-side fluid to flow across the shell to enhance heat transfer and to maintain uniform spacing between the tubes. Despite their widespread use, shell and- tube heat exchangers are not suitable for use in automotive and aircraft applications because of their relatively large size and weight. Note that the tubes in a shell-and-tube heat exchanger open to some large flow areas called *headers* at both ends of the shell, where the tube-side fluid accumulates before entering the tubes and after leaving them. Shell-and-tube heat exchangers are further classified according to the number of shell and tube passes involved. Heat exchangers in which all the tubes make one U-turn in the shell, for example, are called *one-shell-pass and two tube- passes* heat exchangers. Likewise, a heat exchanger that involves two passes in the shell and four passes in the tubes is called a *two-shell-passes and four-tube-passes* heat exchanger.



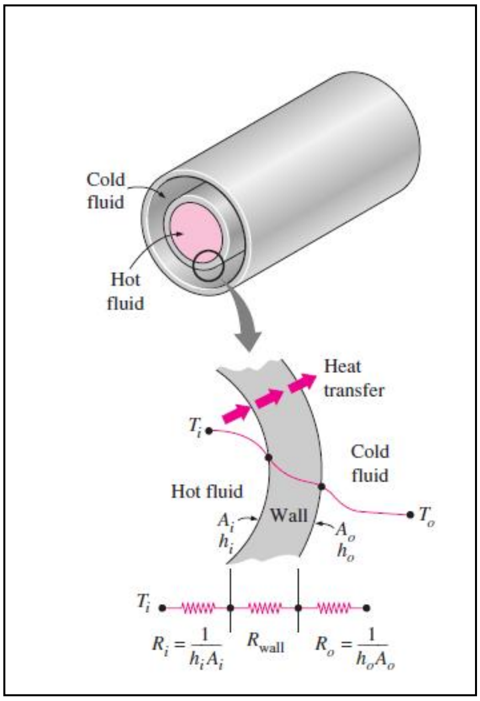
THE OVERALL HEAT TRANSFER COEFFICIENT

A heat exchanger typically involves two flowing fluids separated by a solid wall. Heat is first transferred from the hot fluid to the wall by *convection*, through the wall by *conduction*, The thermal resistance network associated with this heat transfer process involves two convection and one conduction resistances, as shown. Here the subscripts *i* and *o* represent the inner and outer surfaces of the inner tube For a double-pipe heat exchanger, we have $A_i = \pi D_i L$ and $A_o = \pi D_o L$.

$$R_{wall} = \frac{\ln(D_o/D_i)}{2\pi kL}$$

where k is the thermal conductivity of the wall material and L is the length of the tube. Then the *total thermal resistance* becomes

$$R = R_{total} = R_i + R_{wall} + R_o = \frac{1}{h_iA_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{1}{h_oA_o}$$



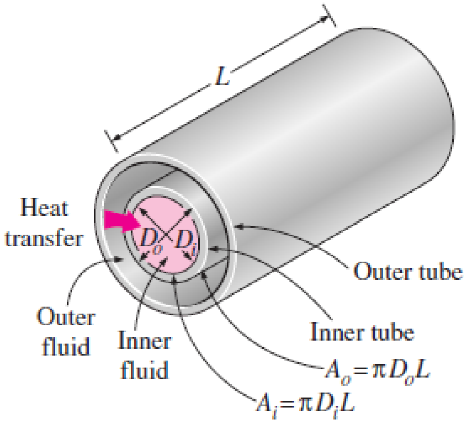
$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R = \frac{1}{h_iA_i} + R_{wall} + \frac{1}{h_oA_o}$$

where the subscripts i and o stand for the inner and outer surfaces of the wall that separates the two fluids, respectively. When the wall thickness of the tube is small and the thermal conductivity of the tube material is high, the last relation simplifies to

$$\frac{1}{U} \approx \frac{1}{h_i} + \frac{1}{h_o}$$

where $U \approx U_i \approx U_o$. The effects of fouling on both the inner and the outer surfaces of the tubes of a heat exchanger can be accounted for by

$$\frac{1}{UA_s} = \frac{1}{U_iA_i} = \frac{1}{U_oA_o} = R$$
$$= \frac{1}{h_iA_i} + \frac{R_{f,i}}{A_i} + \frac{\ln(D_o/D_i)}{2\pi kL} + \frac{R_{f,o}}{A_o} + \frac{1}{h_oA_o}$$



When the tube is *finned* on one side to enhance heat transfer, the total heat transfer surface area on the finned side becomes

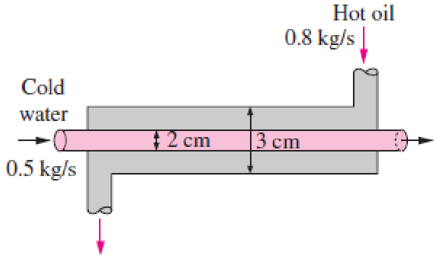
$A_s = A_{total} = A_{fin} = A_{unfinned}$

where A_{fin} is the surface area of the fins and $A_{unfinned}$ is the area of the unfinned portion of the tube surface. For short fins of high thermal conductivity, we can use this total area in the convection resistance relation $R_{conv} = 1/hA_s$ since the fins in this case will be very nearly isothermal. Otherwise, we should determine the effective surface area A from

$A_s = A_{unfinned} = \eta_{fin} A_{fin}$

EXAMPLE 1

Hot oil is to be cooled in a double-tube counter-flow heat exchanger. The copper inner tubes have a diameter of 2 cm and negligible thickness. The inner diameter of the outer tube (the shell) is 3 cm. Water flows through the tube at a rate of 0.5 kg/s, and the oil through the shell at a rate of 0.8 kg/s. Taking the average temperatures of the water and the oil to be 45°C and 80°C, respectively, determine the overall heat transfer coefficient of this heat exchanger.



Nusselt number for fully developed laminar flow in a circular annulus with one surface insulated and the other isothermal (Kays and Perkins, Ref. 8.)

D_i/D_o	Nu_i	Nu_o
0.00	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

ANALYSIS OF HEAT EXCHANGERS

In upcoming sections, we will discuss the two methods used in the analysis of heat exchangers. Of these, the *log mean temperature difference* (or LMTD) method is best suited for the first task and the *effectiveness–NTU* method for the second task as just stated. But first we present some general considerations.

The *first law of thermodynamics* requires that the rate of heat transfer from the hot fluid be equal to the rate of heat transfer to the cold one. That is,

$Q = m_c Cp_c(T_{c,\text{ out}} - T_{c,\text{ in}})$

and

$Q = m_h Cp_h(T_{h,\text{ in}} - T_{h,\text{ out}})$

In heat exchanger analysis, it is often convenient to combine the product of the *mass flow rate* and the *specific heat* of a fluid into a single quantity. This quantity is called the **heat capacity rate** and is defined for the hot and cold fluid streams as

$$C_h = \dot{m}_h C_{ph} \qquad \text{and} \qquad C_c = \dot{m}_c C_{pc}$$

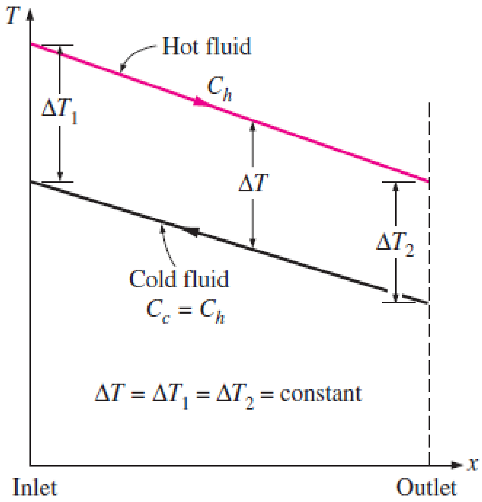


FIGURE 13–12
Two fluids that have the same mass flow rate and the same specific heat experience the change in a well-insulated heat exchanger.

$$\dot{Q} = C_c(T_{c,\text{ out}} - T_{c,\text{ in}})$$

$$\dot{Q} = C_h(T_{h,\text{ in}} - T_{h,\text{ out}})$$

Two special types of heat exchangers commonly used in practice are *condensers* and *boilers*. One of the fluids in a condenser or a boiler undergoes a phase-change process, and the rate of heat transfer is expressed as

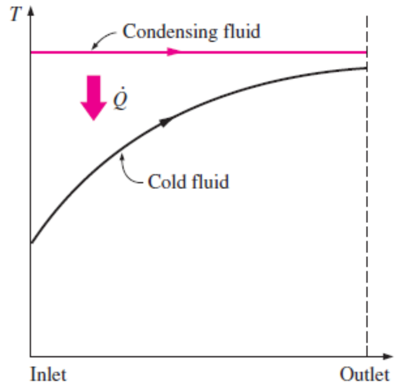
$\dot{Q} = \dot{m} h_{fg}$

where $\dot{m}$ is the rate of evaporation or condensation of the fluid and *hfg* is the enthalpy of vaporization of the fluid at the specified temperature or pressure. An ordinary fluid absorbs or releases a large amount of heat essentially at constant temperature during a phase-change process, as shown in Figure . The heat capacity rate of a fluid during a phase-change process must approach infinity since the temperature change is practically zero. That is,

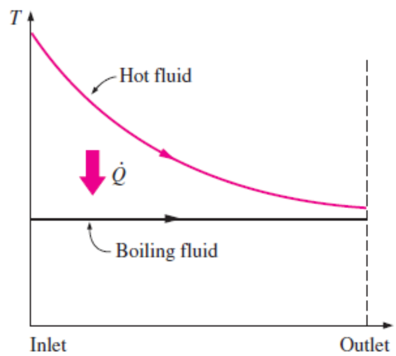
$C = \dot{m} Cp \to \infty$ when $\Delta T \to 0$, so that the heat transfer rate $\dot{Q} = \dot{m} Cp\ \Delta T$ is a finite quantity. Therefore, in heat exchanger analysis, a condensing or boiling fluid is conveniently modeled as a fluid whose heat capacity rate is *infinity*.

The rate of heat transfer in a heat exchanger can also be expressed in an analogous manner to Newton's law of cooling as

$\dot{Q} = UA_s\ \Delta T_m$



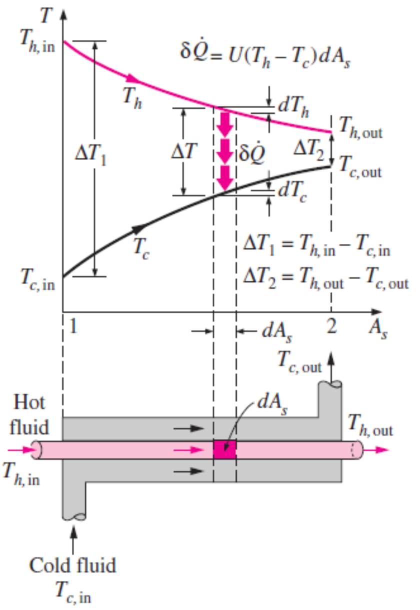
(*a*) Condenser ($C_h \to \infty$)



(*b*) Boiler ($C_c \to \infty$)

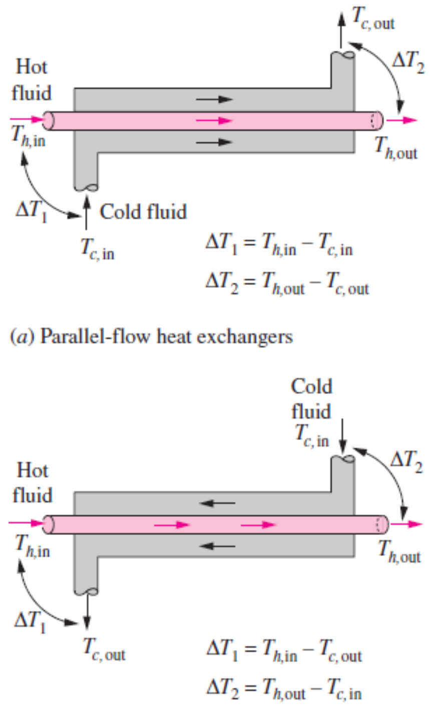
THE LOG MEAN TEMPERATURE DIFFERENCE METHOD

The **log mean temperature difference,** which is the suitable form of the average temperature difference for use in the analysis of heat exchangers. Here ΔT_1 and ΔT_2 represent the temperature difference between the two fluids at the two ends (inlet and outlet) of the heat exchanger .



Counter-Flow Heat Exchangers

The relation above for the log mean temperature difference is developed using a parallel-flow heat exchanger, but we can show by repeating the analysis above for a counter-flow heat exchanger that is also applicable to counter flow heat exchangers. But this time, ΔT_1 and ΔT_2 are expressed as shown in Figure .



(*a*) Parallel-flow heat exchangers

(*b*) Counter-flow heat exchangers

The ΔT_1 and ΔT_2 expressions in parallel-flow and counter-flow heat exchangers.

Multipass and Cross-Flow Heat Exchangers:

Use of a Correction Factor

The log mean temperature difference ΔT_{lm} relation developed earlier is limited to parallel-flow and counter-flow heat exchangers only. Similar relations are also developed for **cross-flow** and **multipass shell-and-tube** heat exchangers, but the resulting expressions are too complicated because of the complex flow conditions.

$\Delta T_{lm} = F\,\Delta T_{lm,\,CF}$

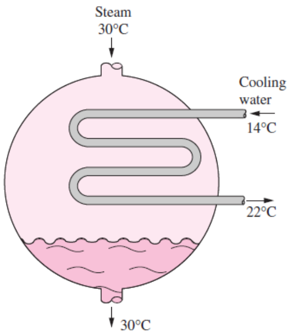
where F is the **correction factor,** which depends on the *geometry* of the heat exchanger and the inlet and outlet temperatures of the hot and cold fluid streams. The correction factor F for common cross-flow and shell-and-tube heat exchanger configurations is given in Figure 13-18 versus two temperature ratios P and R defined as .where the subscripts 1 and 2 represent the *inlet* and *outlet,* respectively. Note that for a shell-and-tube heat exchanger, T and t represent the *shell-* and *tube-side* temperatures, respectively

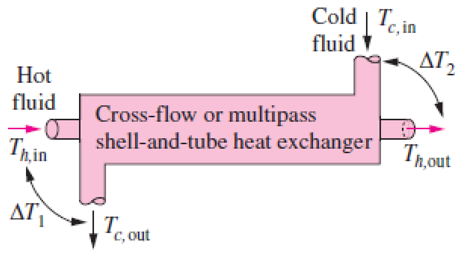
$$P = \frac{t_2 - t_1}{T_1 - t_1}$$

$$R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{(\dot{m}C_p)_{\text{tube side}}}{(\dot{m}C_p)_{\text{shell side}}}$$

EXAMPLE 2 **The Condensation of Steam in a Condenser**

Steam in the condenser of a power plant is to be condensed at a temperature of 30°C with cooling water from a nearby lake, which enters the tubes of the condenser at 14°C and leaves at 22°C. The surface area of the tubes is 45 m^2, and the overall heat transfer coefficient is 2100 W/m^2 · °C. Determine the mass flow rate of the cooling water needed and the rate of condensation of the steam in the condenser.





Heat transfer rate:

$$\dot{Q} = UA_s F \Delta T_{lm,CF}$$

where
$$\Delta T_{\text{lm},CF} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

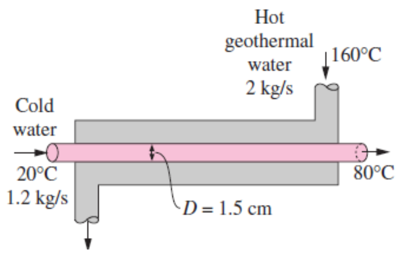
$$\Delta T_1 = T_{h,\text{in}} - T_{c,\text{out}}$$
$$\Delta T_2 = T_{h,\text{out}} - T_{c,\text{in}}$$

and $F = \ldots$ (Fig. 13–18)

The determination of the heat transfer rate for cross-flow and multipass shell-and-tube heat exchangers using the correction factor.

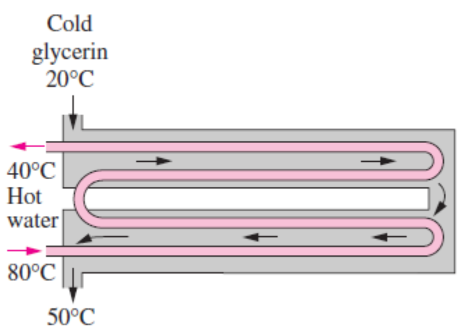
EXAMPLE 3 **Heating Water in a Counter-Flow Heat Exchanger**

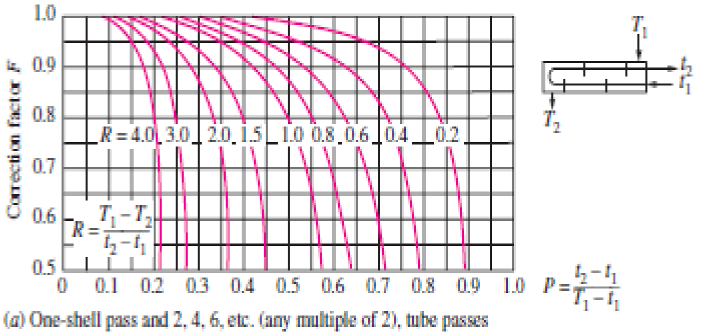
A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a rate of 1.2 kg/s. The heating is to be accomplished by geothermal water available at 160°C at a mass flow rate of 2 kg/s. The inner tube is thin-walled and has a diameter of 1.5 cm. If the overall heat transfer coefficient of the heat exchanger is 640 W/m^2 · °C, determine the length of the heat exchanger required to achieve the desired heating.



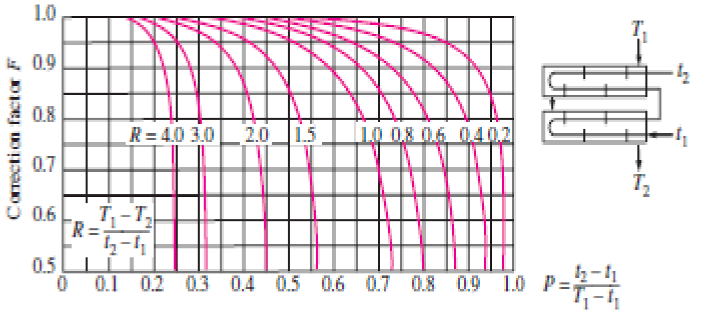
EXAMPLE 4 **Heating of Glycerin in a Multipass Heat Exchanger**

A 2-shell passes and 4-tube passes heat exchanger is used to heat glycerin from 20°C to 50°C by hot water, which enters the thin-walled 2-cm-diameter tubes at 80°C and leaves at 40°C . The total length of the tubes in the heat exchanger is 60 m. The convection heat transfer coefficient is 25 W/m^2 · °C on the glycerin (shell) side and 160 W/m^2 · °C on the water (tube) side. Determine the rate of heat transfer in the heat exchanger (*a*) before any fouling occurs and (*b*) after fouling with a fouling factor of 0.0006 m^2 · °C/ W occurs on the outer surfaces of the tubes.

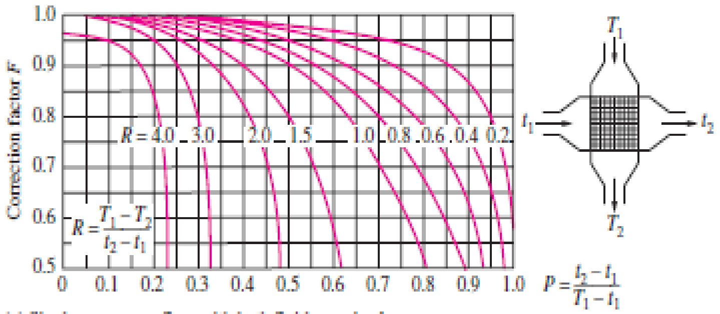




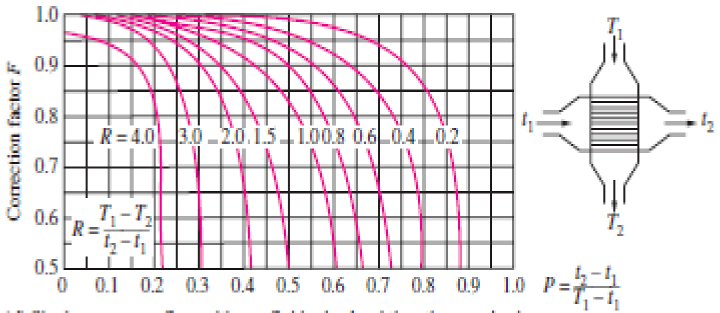
(*a*) One-shell pass and 2, 4, 6, etc. (any multiple of 2), tube passes



(*b*) Two-shell passes and 4, 8, 12, etc. (any multiple of 4), tube passes



(*c*) Single-pass cross-flow with both fluids *unmixed*



(*d*) Single-pass cross-flow with one fluid *mixed* and the other *unmixed*

FIGURE 13–18
Correction factor F charts for common shell-and-tube and cross-flow heat exchangers (from Bowman, Mueller, and Nagle, Ref. 2).

THE EFFECTIVENESS–NTU METHOD

The **effectiveness–NTU method,** which greatly simplified heat exchanger analysis.
This method is based on a dimensionless parameter called the **heat transfer effectiveness** ε , defined as

$$\varepsilon = \frac{\dot{Q}}{\dot{Q}_{max}} = \frac{\text{Actual heat transfer rate}}{\text{Maximum possible heat transfer rate}}$$

The *actual* heat transfer rate in a heat exchanger can be determined from an energy balance on the hot or cold fluids and can be expressed as

$$\dot{Q} = C_c(T_{c,\,out} - T_{c,\,in}) = C_h(T_{h,\,in} - T_{h,\,out})$$

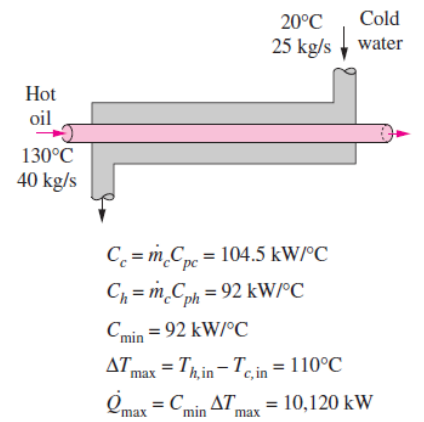
To determine the maximum possible heat transfer rate in a heat exchanger, we first recognize that the *maximum temperature difference* in a heat exchanger is the difference between the *inlet* temperatures of the hot and cold fluids. That is,

$$\Delta T_{max} = T_{h,\,in} - T_{c,\,in}$$

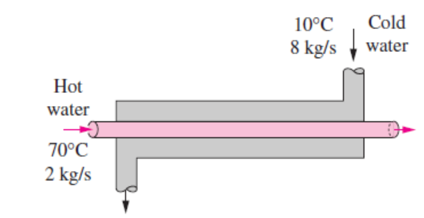
The maximum possible heat transfer rate in a heat exchanger is

$Q_{max} = C_{min}(T_{h,\,in} - T_{c,\,in})$

where C_{min} is the smaller of $C_h = m_h C_{ph}$ and $C_c = m_c C_{pc}$. This is further clarified by the following example. See fig



EXAMPLE 5 **Upper Limit for Heat Transfer in a Heat Exchanger**
Cold water enters a counter-flow heat exchanger at 10°C at a rate of 8 kg/s, where it is heated by a hot water stream that enters the heat exchanger at 70°C at a rate of 2 kg/s. Assuming the specific heat of water to remain constant at Cp = 4.18 kJ/kg · °C, determine the maximum heat transfer rate and the outlet temperatures of the cold and the hot water streams for this limiting case.



The determination of Q max requires the availability of the *inlet temperature* of the hot and cold fluids and their *mass flow rates*, which are usually specified. Then, once the effectiveness of the heat exchanger is known, the actual heat transfer rate Q can be determined from

$\dot{Q} = \varepsilon\, Q_{max} = \varepsilon\, C_{min}(T_{h,\,in} - T_{c,\,in})$

The effectiveness of a heat exchanger depends on the *geometry* of the heat exchanger as well as the *flow arrangement.* Therefore, different types of heat exchangers have different effectiveness relations. Below we illustrate the development of the effectiveness ε relation for the double-pipe *parallel-flow* heat exchanger.

Effectiveness relations of the heat exchangers typically involve the *dimensionless* group $UA_s / C_{\min}$. This quantity is called the **number of transfer units NTU** and is expressed as

$$\text{NTU} = \frac{UA_s}{C_{\min}} = \frac{UA_s}{(\dot{m}C_p)_{\min}}$$

where U is the overall heat transfer coefficient and A_s is the heat transfer surface area of the heat exchanger. Note that NTU is proportional to A_s. Therefore, for specified values of U and $C_{\min}$, the value of NTU *is a measure of the heat transfer surface area* A_s. Thus, the larger the NTU, the larger the heat exchanger.

In heat exchanger analysis, it is also convenient to define another dimensionless quantity called the **capacity ratio** c as

$$c = \frac{C_{\min}}{C_{\max}}$$

It can be shown that the effectiveness of a heat exchanger is a function of the number of transfer units NTU and the capacity ratio c. That is,

$$\varepsilon = \text{function}\,(UA_s/C_{\min}, C_{\min}/C_{\max}) = \text{function}\,(\text{NTU}, c)$$

Effectiveness relations have been developed for a large number of heat exchangers, and the results are given in Table A

TABLE A

Effectiveness relations for heat exchangers: $\text{NTU} = UA_s/C_{\min}$ and $c = C_{\min}/C_{\max} = (\dot{m}C_p)_{\min}/(\dot{m}C_p)_{\max}$ (Kays and London, Ref. 5.)

Heat exchanger type	Effectiveness relation
1 *Double pipe:*	
Parallel-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 + c)]}{1 + c}$
Counter-flow	$\varepsilon = \dfrac{1 - \exp[-\text{NTU}(1 - c)]}{1 - c\exp[-\text{NTU}(1 - c)]}$
2 *Shell and tube:*	
One-shell pass 2, 4, . . . tube passes	$\varepsilon = 2\left\{1 + c + \sqrt{1 + c^2}\,\dfrac{1 + \exp[-\text{NTU}\sqrt{1 + c^2}]}{1 - \exp[-\text{NTU}\sqrt{1 + c^2}]}\right\}^{-1}$
3 *Cross-flow (single-pass)*	
Both fluids unmixed	$\varepsilon = 1 - \exp\left\{\dfrac{\text{NTU}^{0.22}}{c}[\exp(-c\,\text{NTU}^{0.78}) - 1]\right\}$
$C_{\max}$ mixed, $C_{\min}$ unmixed	$\varepsilon = \dfrac{1}{c}(1 - \exp\{1 - c[1 - \exp(-\text{NTU})]\})$
$C_{\min}$ mixed, $C_{\max}$ unmixed	$\varepsilon = 1 - \exp\left\{-\dfrac{1}{c}[1 - \exp(-c\,\text{NTU})]\right\}$
4 *All heat exchangers with* $c = 0$	$\varepsilon = 1 - \exp(-\text{NTU})$

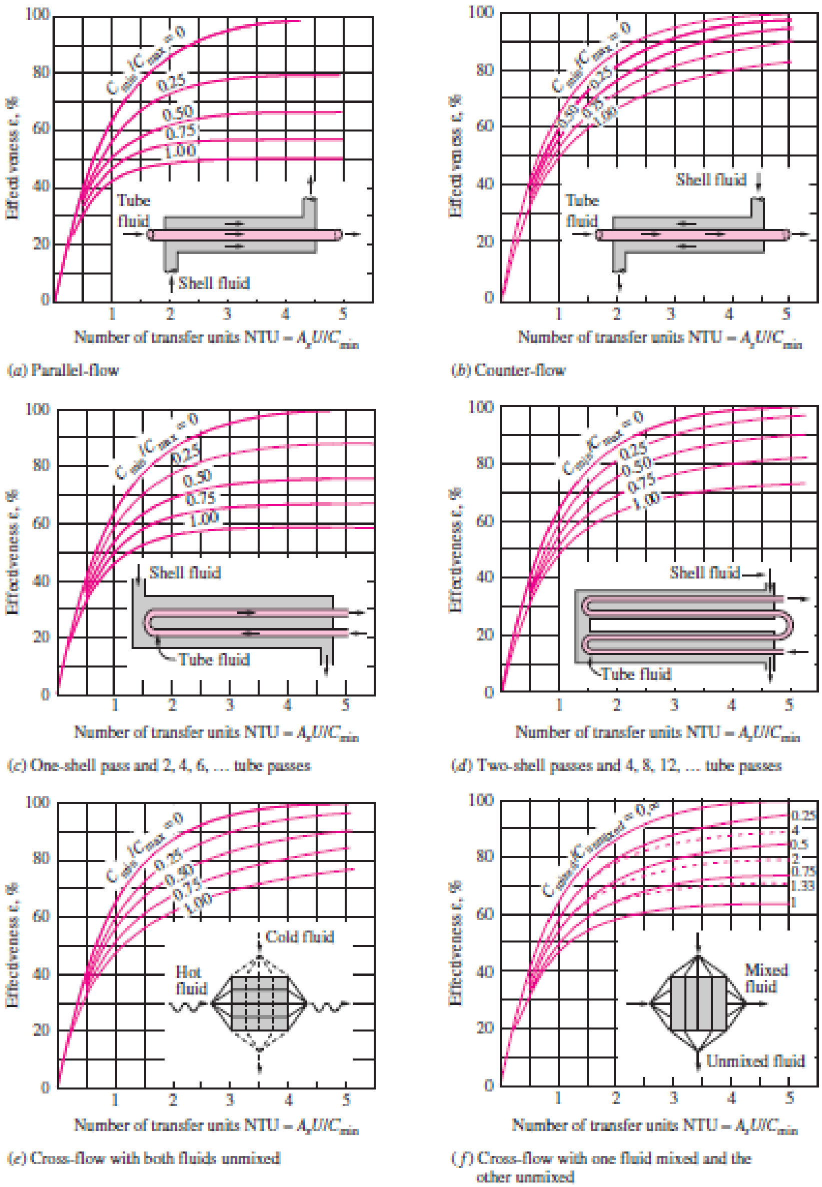


FIGURE 13–26

Effectiveness for heat exchangers (from Kays and London, Ref. 5).

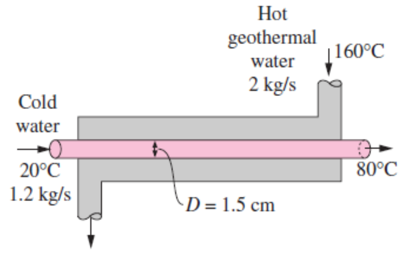
TABLE B NTU relation

NTU relations for heat exchangers NTU = UA_s/C_{min} and $c = C_{min}/C_{max} = (\dot{m}C_p)_{min}/(\dot{m}C_p)_{max}$ (Kays and London, Ref. 5.)

Heat exchanger type	NTU relation
1 *Double-pipe:* Parallel-flow	$\text{NTU} = -\dfrac{\ln[1 - \varepsilon(1 + c)]}{1 + c}$
Counter-flow	$\text{NTU} = \dfrac{1}{c - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon c - 1}\right)$
2 *Shell and tube:* One-shell pass 2, 4, . . . tube passes	$\text{NTU} = -\dfrac{1}{\sqrt{1 + c^2}}\ln\left(\dfrac{2/\varepsilon - 1 - c - \sqrt{1 + c^2}}{2/\varepsilon - 1 - c + \sqrt{1 + c^2}}\right)$
3 *Cross-flow (single-pass)* C_{max} mixed, C_{min} unmixed	$\text{NTU} = -\ln\left[1 + \dfrac{\ln(1 - \varepsilon c)}{c}\right]$
C_{min} mixed, C_{max} unmixed	$\text{NTU} = -\dfrac{\ln[c\ln(1 - \varepsilon) + 1]}{c}$
4 *All heat exchangers with* $c = 0$	$\text{NTU} = -\ln(1 - \varepsilon)$

EXAMPLE 6 **Using the Effectiveness–NTU method**

Repeat Example 3, which was solved with the LMTD method, using the effectiveness–NTU method.



EXAMPLE 7 **Cooling Hot Oil by Water in a Multipass Heat Exchanger**

Hot oil is to be cooled by water in a 1-shell-pass and 8-tube-passes heat exchanger. The tubes are thin-walled and are made of copper with an internal diameter of 1.4 cm. The length of each tube pass in the heat exchanger is 5 m, and the overall heat transfer coefficient is 310 W/m^2 · °C. Water flows through the tubes at a rate of 0.2 kg/s, and the oil through the shell at a rate of 0.3 kg/s. The water and the oil enter at temperatures of 20°C and 150°C, respectively. Determine the rate of heat transfer in the heat exchanger and the outlet temperatures of the water and the oil.