History of Logarithms:

Logarithms were invented independently by John Napier, a Scotsman, and by Joost Burgi, a Swiss. The logarithms which they invented differed from each other and from the common and natural logarithms now in use. Napier's logarithms were published in 1614; Burgi's logarithms were published in 1620. The objective of both men was to simplify mathematical calculations. Napier's approach was algebraic and Burgi's approach was geometric. Neither men had a concept of a logarithmic base. Napier defined logarithms as a ratio of two distances in a geometric form, as opposed to the current definition of logarithms as exponents. The possibility of defining logarithms as exponents was recognized by John Wallis in 1685 and by Johann Bernoulli in 1694.

The invention of the common system of logarithms is due to the combined effort of Napier and Henry Biggs in 1624. Natural logarithms first arose as more or less accidental variations of Napier's original logarithms. Their real significance was not recognized until later. The earliest natural logarithms occur in 1618.

Logarithms are useful in many fields from finance to astronomy.

Exponential Functions

In this section we're going to review one of the more common functions in both calculus and the sciences. However, before getting to this function let's take a much more general approach to things.

Let's start with $b > 0$, $b \neq 1$. An exponential function is then a function in the form,

$$f(x) = b^x$$

Note that we avoid $b = 1$ because that would give the constant function, $f(x) = 1$. We avoid $b = 0$ since this would also give a constant function and we avoid negative values of *b* for the following reason. Let's, for a second, suppose that we did allow *b* to be negative and look at the following function.

$$g(x) = (-4)^x$$

Let's do some evaluation.

$$g(2) = (-4)^2 = 16 \qquad\qquad g\left(\frac{1}{2}\right) = (-4)^{\frac{1}{2}} = \sqrt{-4} = 2i$$

So, for some values of *x* we will get real numbers and for other values of *x* we well get complex numbers. We want to avoid this and so if we require $b > 0$ this will not be a problem.

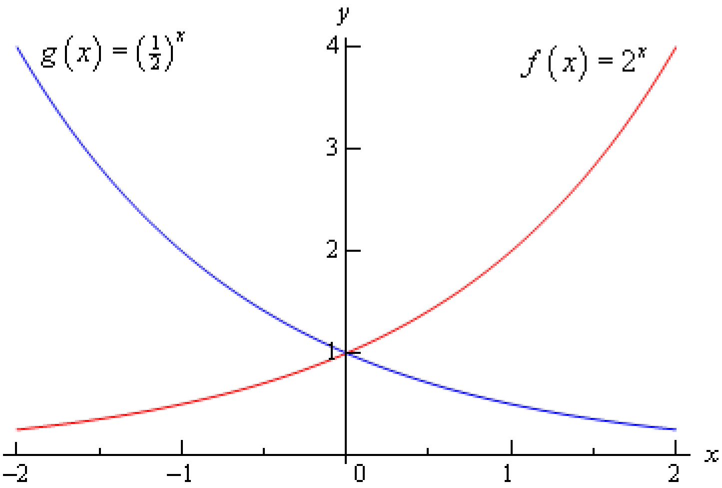
Let's take a look at a couple of exponential functions.

Example 1 Sketch the graph of $f(x) = 2^x$ and $g(x) = \left(\frac{1}{2}\right)^x$

Solution
Let's first get a table of values for these two functions.

x	*f(x)*	*g(x)*
-2	$f(-2) = 2^{-2} = \frac{1}{4}$	$g(-2) = \left(\frac{1}{2}\right)^{-2} = 4$
-1	$f(-1) = 2^{-1} = \frac{1}{2}$	$g(-1) = \left(\frac{1}{2}\right)^{-1} = 2$
0	$f(0) = 2^0 = 1$	$g(0) = \left(\frac{1}{2}\right)^0 = 1$
1	$f(1) = 2$	$g(1) = \frac{1}{2}$
2	$f(2) = 4$	$g(2) = \frac{1}{4}$

Here's the sketch of both of these functions.



This graph illustrates some very nice properties about exponential functions in general.

Properties of $f(x) = b^x f(x) = b^x$

1. $f(0) = 1$. The function will always take the value of 1 at $x = 0$.
2. $f(x) \neq 0$. An exponential function will never be zero.
3. $f(x) > 0$. An exponential function is always positive.
4. The previous two properties can be summarized by saying that the range of an exponential function is $(0, \infty)$.
5. The domain of an exponential function is $(-\infty, \infty)$. In other words, you can plug every *x* into an exponential function.
6. If $0 < b < 1$ then,
   a. $f(x) \to 0$ as $x \to \infty$
   b. $f(x) \to \infty$ as $x \to -\infty$
7. If $b > 1$ then,
   a. $f(x) \to \infty$ as $x \to \infty$ $f(x) \to 0$ as $x \to -\infty$

These will all be very useful properties to recall at times as we move throughout this course (and later Calculus courses for that matter…).

There is a very important exponential function that arises naturally in many places. This function is called the **natural exponential function**. However, for most people this is simply the exponential function.

Definition : The **natural exponential function** is $f(x) = \mathbf{e}^x$ where, $\mathbf{e} = 2.71828182845905\ldots$

So, since $\mathbf{e} > 1$ we also know that $\mathbf{e}^x \to \infty$ as $x \to \infty$ and $\mathbf{e}^x \to 0$ as $x \to -\infty$.

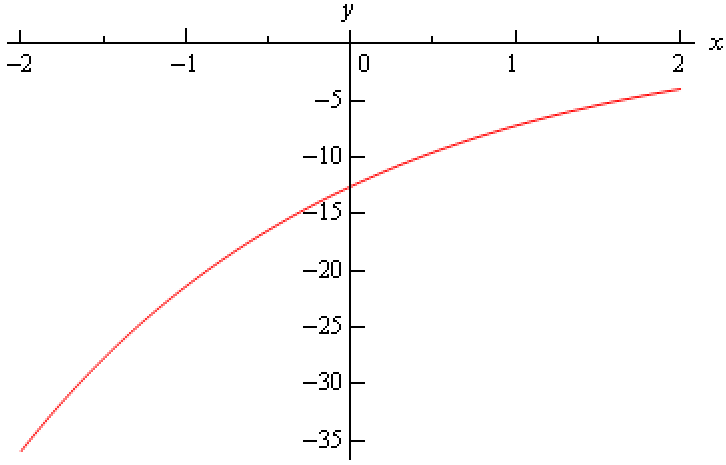
- Let's take a quick look at an example.
- ***Example 2*** Sketch the graph of $h(t) = 1 - 5e^{1-\frac{t}{2}}$

Solution

Let's first get a table of values for this function.

t	-2	-1	0	1	2	3
$h(t)$	-35.9453	-21.4084	-12.5914	-7.2436	-4	-2.0327

Here is the sketch.



The main point behind this problem is to make sure you can do this type of evaluation so make sure that you can get the values that we graphed in this example.