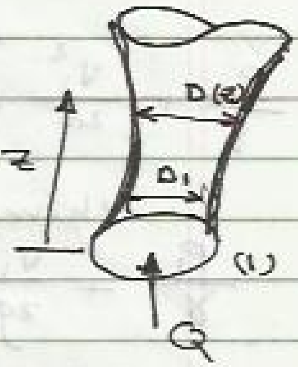
EX Water flows upward through a variable area pipe with a constant flowrate (Q), as shown in fig. If viscous effects are negligible, determine the diameter D(z), in terms of D_1, if the pressure is to remain constant throughout the pipe. That is $p(z) = P_1$

Sol

$$\cancel{\frac{p}{\gamma}} + \frac{V^2}{2g} + z = \cancel{\frac{P_1}{\gamma}} + \frac{V_1^2}{2g} + \overset{0}{\cancel{z_1}}$$



with $p = P_1$ and $z_1 = 0$

$$\frac{V^2}{2g} + z = \frac{V_1^2}{2g}$$

$$V_1^2 - V^2 = 2gz \quad \text{——} ①$$

$$Q_1 = A_1 V_1 \Rightarrow V_1 = \frac{Q}{A_1} = \frac{4Q}{\pi D_1^2}$$

and

$$Q = AV \Longrightarrow V_z = \frac{Q}{A} = \frac{4Q}{\pi D_{(z)}^2}$$

sub in ①

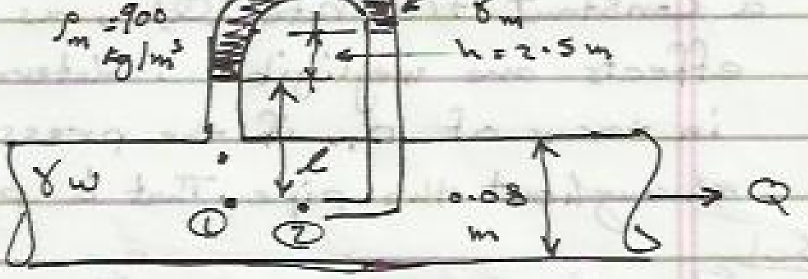
$$\left(\frac{4Q}{\pi D_1^2}\right)^2 - \left(\frac{4Q}{\pi D_{(z)}^2}\right)^2 = 2(9.81) * z \quad] * \frac{\pi^2}{16Q^2}$$

$$\frac{1}{D_1^4} - \frac{1}{D(z)^4} = \frac{\pi^2 g z}{8Q^2}$$

Thus

$$D_{(z)} = \frac{D_1}{\left[1 - \frac{\pi^2 g D_1^4 z}{8Q^2}\right]^{1/4}} \qquad \underline{\underline{\text{ans.}}}$$

<u>EX</u> Determine the flowrate through the pipe in fig?



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \overset{0}{\cancel{z_1}} = \frac{P_2}{\gamma} + \frac{\overset{0}{\cancel{V_2^2}}}{2g} + \cancel{z_2}$$

where $z_1 = z_2$ and $V_2 = 0$

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma}$$

$$\therefore\ V_1 = \sqrt{2g\left(\frac{P_2 - P_1}{\gamma}\right)} \quad \text{— ①}$$

but

$$P_1 - \gamma l - \gamma_m h + \gamma(l + h) = P_2 \Rightarrow P_1 - \cancel{\gamma l} - \gamma_m h + \cancel{\gamma l} + \gamma h = P_2$$

$$\therefore\ P_2 - P_1 = (\gamma - \gamma_m)h$$

so that

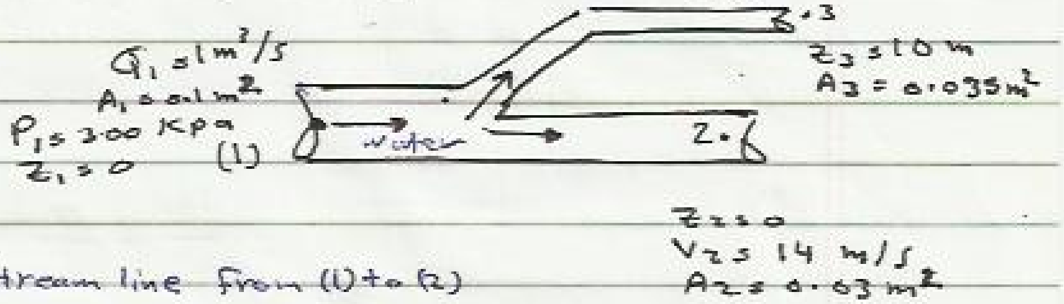
$$V_1 = \sqrt{2g\,\frac{(\gamma - \gamma_m)h}{\gamma}} = \sqrt{2g\left(1 - \frac{\gamma_m}{\gamma}\right)h}$$

$$= \sqrt{2 \times 9.81^{m/s^2} \times \cancel{\gamma}\left(1 - \frac{900\ ^{kg/m^3}}{1000\ ^{kg/m^3}}\right)(2.5)}$$

$$= 2.2\ \frac{m}{s}$$

$$\therefore\ Q = A_1 V_1 = \frac{\pi}{4}(0.08)^2(2.2) = 0.011\ m^3/s$$

Ex Water Flows through the branching pipe shown in fig. If viscous effects are negligible. Determine the pressure at section (2) and the pressure at section (3).



Sol along the stream line from (1) to (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \overset{0}{\cancel{Z_1}} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \overset{0}{\cancel{Z_2}}$$

$$V_1 = \frac{Q}{A_1} = \frac{1 \, m^3/s}{0.1 \, m^2} = 10 \, m/s$$

Thus

$$\frac{300 \times 10^3}{9.81 \times 1000} + \frac{(10)^2}{2 * 9.81} = \frac{P_2}{9.81 \times 1000} + \frac{(14)^2}{2 * 9.81}$$

$$\therefore P_2 = 2.52 \times 10^5 \, N/m^2 = 252 \, kpa$$

$$Q_2 = A_2 V_2 = 14 \times 0.03$$

along the streamline from (1) to (3)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \overset{0}{\cancel{Z_1}} = \frac{P_3}{\gamma} + \frac{V_3^2}{2g} + Z_3$$

$$Q_1 = Q_2 + Q_3 \Rightarrow Q_3 = Q_1 - Q_2$$

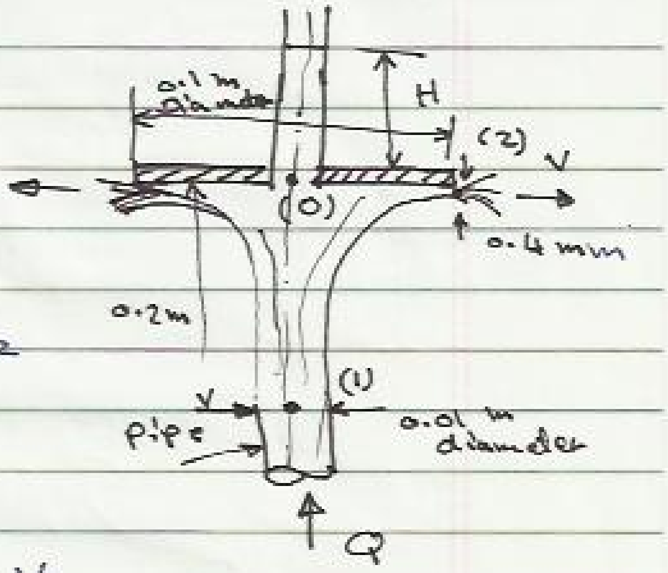
$$A_3 V_3 = Q - A_2 V_2$$

$$\therefore V_3 = \frac{Q_1 - Q_2}{A_3} = \frac{1 - 14 * 0.03}{0.035} = 16.6 \, m/s$$

$$\frac{300 * 10^3}{9.81 * 1000} + \frac{(10)^2}{2 * 9.81} = \frac{P_3}{9.8 * 1000} + \frac{(16.6)^2}{2 * 9.81} + 10$$

$$\therefore P_3 = 1.14 * 10^5 \, N/m^2 = 114 \, Kpa$$

EX water flows from the pipe shown in fig. as free jet and strikes a circular flat plate. the flow geometry shown is axisymmetrical. Determine the flow rate and the manometer reading H.



Sol.

$$\frac{\cancel{p_1}}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} = \frac{\cancel{p_2}}{\gamma} + \frac{V_2^2}{2g} + z_2$$

where

$p_1 = 0,\ p_2 = 0,\ z_1 = 0,\ z_2 = 0.2$

thus

$$\frac{V_1^2}{2g} = \frac{V_2^2}{2g} + z_2 \longrightarrow ①$$

$$\because Q = A_1V_1 = A_2V_2 \Rightarrow V_1 = \frac{A_2}{A_1}V_2$$

$$V_1 = \frac{\pi \overset{2}{D}_2 h\, V_2}{\frac{\pi}{4} D_1^2} = \frac{4 D_2 h}{D_1^2} V = \frac{4(0.1m)(4*10^{-4}m)}{(0.01)^2} V_2$$

$$= 1.6V_2 = 1.6V$$

hence eq ① gives

$$V_1^2 = V_2^2 + 2g z_2$$

$$(1.6V_2)^2 = V_2^2 + 2(9.81)(0.2) \Rightarrow V_2 = 1.59\ m/s$$

$$\therefore Q = A_2V_2 = \pi(0.1\,m)(4*10^{-4}\,m)(1.59\ m/s) = \underline{\underline{2*10^{-4}\ \frac{m^3}{s}}}\ \text{ans}$$

H

Also

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \boxed{\frac{p_0}{\gamma}} + \frac{V_0^2}{2g} + z_0$$

where $V_0 = 0$, $z_0 = 0.2$, $V_1 = 1.6V_2$

$V_1 = 1.6(1.59) = 2.54\ m/s$ and $p_1 = 0$

$$H = \frac{p_0}{\gamma} = \frac{V_1^2}{2g} - z_0 = \frac{(2.54\ m/s)^2}{2(9.81\ m/s^2)} - 0.2 = 0.129\ m$$