The Buckingham π-Theorem :

"If a physical phenomenon involves (n) quantities and if these quantities involves (m) dimensions, then the quantities can be arranged into (n-m) dimensionless parameters."

let $(A_1, A_2, \ldots, A_n)$ ~~to~~ be quantities involved in a physical problem. A functional relationship should exist such that:

$$F(A_1, A_2, \ldots A_n) = 0.0 \quad (1)$$

If the number of dimensions in the above (n) quantities is (m), than there exists (n-m) dimensionless parameters called (π) such that.

$$f(\pi_1, \pi_2, \ldots, \pi_{n-m}) = 0.0 \quad (2)$$

where each (π) term is a non-dimensional product involving the above quantities.

The Method of determining the π-parameter is to select (m) of the "A" quantities with different dimensions that containing among them the (m) dimensions and use them as " ~~rep~~ " repeating Variable" together with one of the other "A" quantities For each (π). Thus

$$\left.\begin{aligned} \pi_1 &= A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4 \\ \pi_2 &= A_1^{x_2} A_2^{y_2} A_3^{z_2} A_5 \\ &\vdots \\ \pi_{n-m} &= A_1^{x_{n-m}} A_2^{y_{n-m}} A_3^{z_{n-m}} A_n \end{aligned}\right\} A_1, A_2, A_3 \text{ repeating Variable}$$

The exponents are to be determined such that (π) is dimensionless.

If only two dimension are involved, then two of A quantities are selected as repeating Variables

Variables in Fluid mechanics are classified as:

① Variables describing geometry of the system, such as length, area, Volume, inclination... etc.

② Variables decribing kinematic and dynamic aspects of flow, such as velocity, velocity gradient, acceleration, pressure, momentum, energy, power, ... etc.

③ Variables decribing the phsical properties of the Fluid such as density, viscosity, specific weight, surface tension, elasticity and vapor pressure.

The procedure For the application of the π - Theorem is

① slect the (n) Variables involved in the problem.

② Write the Functional relationship $F(A_1, A_2, \ldots A_n) = 0.0$

③ Identify the dependent Variable.

④ Select the repeating Variables.

a) They must contain all dimensions of the problems collectively

b) Each repeated Variable must have different dimensions From the other.

c) They must be physically the most significant Variables

EX

we choose the (L) not (D) because its effect is larger.

d) They must not contain least dimensions (ex: (V, g), choose V)

e) They must not contain the dependent Variable.

F) They should not form a π - term by themselves.

g) It is essential that no one of the repeating Variables be ~~derived~~ derivable from the other repeating Variable.

h) A variable of minor importance should not be selected as repeating Variable.

⑤ write the π-parameters in terms of unknown exponent

$\pi_1 = A_1^{x_1} A_2^{y_1} A_3^{z_1} A_4$

⑥ For each π-expression write the equations of the exponent so that the sum of the exponents of each dimension will be zero.

⑦ solve the equations simultaneously.

⑧ substitute back into the the π-expression of step ⑤ the exponents to obtain the dimensionless π-parameters

⑨ Establish the functional relation

$f(\pi_1, \pi_2, \pi_3, \ldots \pi_{n-m})$

or solve for one of the π's explicitly

$\pi_2 = f_1(\pi_1, \pi_3, \pi_4, \ldots)$

⑩ Recombine, if desired, to after the forms of the π-parameters, keeping the same number of independent parameters.

EX $F_D = f(L, V, \rho, \mu, k)$

sol $F(F_D, L, V, \rho, \mu, k) = 0.0$

$\left.\begin{array}{l} n = 6 \\ m = 3 \end{array}\right\} \Rightarrow$ no of π's $= 3$

let repeatary variable $= (L, V, \rho)$

$\pi_1 = L^{x_1} V^{y_1} \rho^{z_1} F_D$

$M^0 L^0 T^0 = L^{x_1} (LT^{-1})^{y_1} (ML^{-3})^{z_1} MLT^{-2}$

$$\left.\begin{array}{ll} M: & 0 = z_1 + 1 \\ L: & 0 = y_1 - 3z_1 + 1 + x_1 \\ T: & 0 = -y_1 - 2 \end{array}\right\} \Rightarrow \begin{array}{l} x_1 = -2 \\ y_1 = -2 \\ z_1 = -1 \end{array} \Rightarrow \boxed{\pi_1 = \frac{F_D}{\rho L^2 V^2}}$$

$\pi_2 = L^{x_2} V^{y_2} \rho^{z_2} \mu$

$M^0 L^0 T^0 = L^{x_2} (LT^{-1})^{y_2} (ML^{-3})^{z_2} ML^{-1}T^{-1}$

$\therefore \boxed{\pi_2 = \frac{\mu}{\rho L V}}$

$$\pi_3 = L^{x_3} V^{y_3} \rho^{z_3} K$$

$$\Rightarrow \boxed{\pi_3 = \frac{K}{\rho V^2}}$$

$$\therefore f\left(\frac{F_D}{\rho L^2 V^2}, \frac{\mu}{\rho L V}, \frac{K}{\rho V^2} \right) = 0$$

or

$$\frac{F_D}{\rho L^2 V^2} = f'\left(\frac{\mu}{\rho L V}, \frac{K}{\rho V^2} \right)$$

$$\frac{\mu}{\rho L V} = \frac{1}{Re} \quad \text{and} \quad \frac{K}{\rho V^2} = \frac{1}{M}$$

$$\therefore F_D = \rho L^2 V^2 f'\left(\frac{1}{Re}, \frac{1}{M} \right)$$
$$= \rho L^2 V^2 f'(Re, M)$$