or $\quad Log(S_n) = Log(a) + b\ Log(N)$

where

$$b = \frac{1}{Z} Log\left(\frac{S_o}{S_e}\right) \qquad , Z = Log(N_1) - Log(N_2)$$

$$Log(a) = Log(S_o) - 3b$$

For a material with a Knee (for ferrous materials)

$N_2 = 10^6$, $N_1 = 10^3$, $Z = 3-6 = -3$

For a material without Knee (for non-ferrous materials)

$N_2 = 5\times10^8$, $N_1 = 10^3$, $Z = \log(10^3) - \log(5\times10^8) = -5.699$

$$b = -\frac{1}{5.699} \log\left(\frac{S_o}{S_f}\right)$$

Ex. Create an estimation of S–N diagram for a steel, axially loaded bar and define its equation. How many cycles of life can be expected if the alternating stress is 100 MPa. The bar is 150 mm square and has a hot-rolled finish. The operating temp. is 500 °C. $\sigma_{ut} = 600$ MPa. The loading will be pure axial, fully reversed.

► $S_e' \cong 0.5\ \sigma_{ut} = 0.5(600) = 300$ MPa

$C_{load} = 0.7 \quad$ (Axial Load)

The part is larger than the test specimen and is not round

$A_{95} = bh = 150\times150 = 22500\ mm^2$ [For pure axial loading, the stress is uniformly distributed over area]

$$d_{equiv} = \sqrt{\frac{A_{95}}{0.0766}} = 542\ mm$$

$C_{size} = 0.6$

$C_{surface} = A\ \sigma_{ut}^{\,b} = 57.7(600)^{-0.718} = 0.584$

$C_{temp.} = 1 - 0.0058(T - 450) = 1 - 0.0058(500 - 450) = 0.71$

$C_{reliab} = 0.753$ for desired 99.9%

$$S_e = C_{load}\; C_{size}\; C_{surf}\; C_{temp}\; C_{reliab}\; S_e'$$
$$= (0.7)\;(0.6)\;(0.584)\;(0.71)\;(0.753)\;300$$
$$= 39 \text{ MPa}$$

$$S_o = 0.75\,\sigma_{ut} = 0.75\,(600) = 450 \text{ MPa}$$

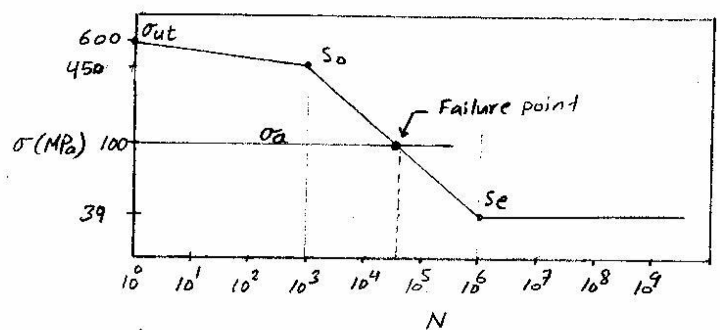
$$b = -\frac{1}{3}\log\left(\frac{S_o}{S_e}\right) = -\frac{1}{3}\log\left(\frac{450}{39}\right) = -0.3539$$

$$\log(a) = \log(S_o) - 3b = \log(450 * 10^6) - 3(-0.3539)$$

$$\Rightarrow a = 5187.93 * 10^6$$

$$S_n = aN^b = (5187.93)\; N^{(-0.3539)} (\text{MPa}),\; 10^3 \leq N \leq 10^6$$

$$S_n = S_e = 39 \text{ MPa}$$



$$S_n = a N^b$$

$$100 = 5187.93\; N^{-0.3539} \qquad \Rightarrow N = 70143. \text{ cycles}$$

Ex. Create an estimated S-N diagram for an aluminum bar and define its equation. What is the corrected fatigue strength at $2 * 10^7$ cycles? $\sigma_{ut} = 310$ MPa, the forged bar is 3.81 cm in round. The load is fully reversed torsion.

► $S_f' \cong 0.4\,\sigma_{ut} \cong (0.4)(310) = 124$ MPa at $N = 5 \times 10^8$ cycles

for pure torsion $C_{load} = 1$

$$C_{size} = 1.189\, d^{-0.097} = 1.189\,(38.1)^{-0.097} = 0.835$$

in (mm)

for forged finish $C_{surf} = A\,\sigma_{ut}^{\,b} = 272\,(310)^{-0.995} = 0.903$

in (MPa)

$C_{temp} = 1$

the reliability is taken at 99 %.

$C_{reliab} = 0.814$

$$\therefore\ S_f = C_{load}\, C_{size}\, C_{surf}\, C_{temp}\, C_{reliab}\, S_f'$$

$$= (1)(0.835)(0.903)(1)(0.814)(124)$$

$$= 76.1062 \text{ MPa}$$

for torsion

$$S_o = 0.9\,\sigma_{ut} = 0.9 \times 310 = 279 \text{ MPa} \quad \text{at } N = 10^3 \text{ cycles}$$

$$S_n = a N^b$$

$$b = -\frac{1}{5.699}\log\left(\frac{279}{76.1062}\right) = -0.09899$$

$$\log(a) = \log(279 \times 10^6) - 3(-0.09899) \Rightarrow a = 552.807 \text{ MPa}$$

at $N = 2 \times 10^7$ cycles

$$S_n = a N^b = 552.807\,(2 \times 10^7)^{-0.09899} = 104.675 \text{ MPa}$$

Notches and Stress Concentration

Notche is a generic term referes to any geometric contour that disrupts the force flow through the part. A notch can be a hole, a groove, a fillet, an abrupt change in cross section. It is assumed that the engineer will follow good design practice and keep the radii of these notches as large as possible and reduce stress concentrations

A notch creats a stress concentration that raises the stresses locally and even cause local yielding. for static loads, the stress concentrations were only of concern for brittle material (as depicted before). with dynamic loads, brittle and ductile materials are both sensitive to the notch.

Neuber and Peterson approches the notch sensitivity "q" as

$$q = \frac{K_f - 1}{K_t - 1} \quad \text{or} \quad K_f = 1 + q\,(K_t - 1)$$

K_t is the static stress concentration factor

K_f " " fatigue (dynamic) stress concentration factor

The nominal dynamic stress for any situation is then increased by the factor K_f, thus

$$\sigma = K_f\, \sigma_{nom} \qquad \tau = K_{f_s}\, \tau_{nom}$$

The notch sensitivity (q) can be defined as

$$q = \frac{1}{1 + \frac{\sqrt{a}}{\sqrt{r}}}$$

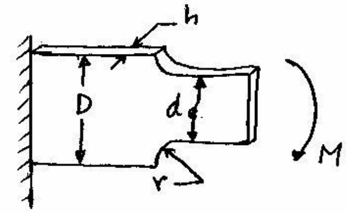
a: Neuber's Constant (From table)

r: notch radius

or using chart for each material

Ex. A rectangular stepped bar is to be loaded in bending.

Determin the fatigue stress concentration factor for a given dimensions. $\sigma_{ut} = 324$ MPa (steel)



$D = 2$ cm , $d = 1.8$ cm

$r = 0.25$ cm

► From Appendix E , Fig E-10

$K_t \cong A\left(\frac{r}{d}\right)^b$

$A = 1.0147$, $b = -0.21793$

$K_t \cong 1.56$

From Fig. 6.36

$q \approx 0.7$

$K_f = 1 + q(K_t - 1) = 1 + 0.7(1.56 - 1) = 1.392$

and the endurance limit

$S_e = C_{load}\ C_{size}\ C_{surf}\ C_{temp}\ C_{reliab}\ S_e'$