Fatigue Failure

Most failures in machinery are due to time-varying loads rather than to static loads. These failures typically occur at stress levels significantly lower than the yield strengths of the materials. Thus, using only the static failure theories can lead to unsafe designs when loads are dynamic.

Mechanism of Fatigue Failure

There are three stages of fatigue failure

1. crack initiation stage

It is occur in a microscopic scale, where some regions have some geometric stress concentration (notches). when the stress at the notch oscillate, local yielding may occur due to the stress concentration. The localized plastic yielding causes distortion and creats slip bands (due to shear motion) along the crystal boundaries of the material. As the stress cycles, additional slip bands occur and initiate microscopic craks.

2. Crack propagation stage

During the cyclic stress, the crack grows in a small amount and the crack surface will open and close where the crack will grow due to tensile stress above the tensile yield of the material at the crack tip.

3. sudden fracture stage

since the remaining material cannot support the loads, a sudden and Fast fracture occur.

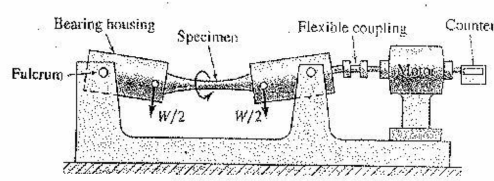
Fatigue-Life Methods

The three major fatigue life methods used in design and analysis are

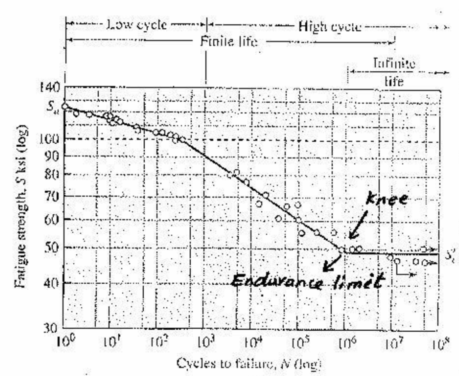
1. stress - life method.
2. strain - life method.
3. Linear - elastic fracture mechanics method.

The stress - Life Method

To establish the fatigue strength of a material, a number of tests are necessary. For a rotating beam test, a constant bending load is applied, and the number of revolutions (stress reversals) of the beam required for failure is recorded. The first test is made at a stress that is somewhat under the ultimate strength of the material. The second test is made at a stress that is less than that used in the first. This process is continued, and the results are plotted as an S-N diagram, where S represent the fatigue strength and N represent the number of cycles. In the case of the steels, a knee occurs in the graph, and beyond this knee failure will not occur, no matter how great the number of cycles. The strength corresponding to the knee is called the "endurance limit", or fatigue limit.



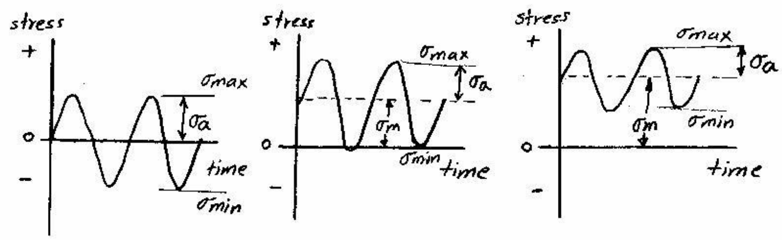
Fatigue life test machine



S-N diagram

Fatigue loads

Any loads that vary with time can potentially cause fatigue failure. The typical stress – time functions experienced by rotating machinery can be modeld as shown in the Fig. below.



Fully reversed Repeated Fluctuating

This Fig. shows the Fully reversed case for which the mean value is zero. The repeated case in which the wave form ranges from zero to a maximum with $\sigma_m = \sigma_a$. And the more general case (called fluctuating stress) with all component values non-zero.

stress range $\Delta\sigma$ is defined as

$$\Delta\sigma = \sigma_{max.} - \sigma_{min}$$

the mean component σ_m is

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

the alternating component σ_a

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Estimation of Theoretical Fatigue strength S_f' or Endurance limit S_e'

From the experment data, an approximate relationships can be stated between σ_{ut} and S_f' or S_e'.

steels ($N=10^6$)	$S_e' \cong 0.5\,\sigma_{ut}$	for $\sigma_{ut} < 1400$ MPa (200 Ksi)
	$S_e' \cong 700$ MPa (100 Ksi)	for $\sigma_{ut} \geq 1400$ MPa (200 Ksi)
Irons ($N=10^6$)	$S_e' \cong 0.4\,\sigma_{ut}$	for $\sigma_{ut} < 400$ MPa (60 Ksi)
	$S_e' \cong 160$ MPa (24 Ksi)	for $\sigma_{ut} \geq 400$ MPa (60 Ksi)
Aluminum ($N=5\times10^8$)	$S_f' \cong 0.4\,\sigma_{ut}$	for $\sigma_{ut} < 330$ MPa (48 Ksi)
	$S_f' \cong 130$ MPa (19 Ksi)	for $\sigma_{ut} \geq 330$ MPa (48 Ksi)
copper ($N=5\times10^8$)	$S_f' \cong 0.4\,\sigma_{ut}$	for $\sigma_{ut} < 280$ MPa (40 Ksi)
	$S_f' \cong 100$ MPa (14 Ksi)	for $\sigma_{ut} \geq 280$ MPa (40 Ksi)

Correction factors to the Theoretical S_f or S_e

since the theoretical S_f' or S_e' is obtained from standard tests specimens and conditions, so it must be modified or corrected to account for physical differences between the test specimen and the actual part being designed.

Environmental and temp. differences between the test conditions and the actual conditions must be taken into account. Differences in the manner of loading need to be account for.

So, the corrected fatigue strength S_f or endurance limit S_e becomes

$$S_f = C_{load}\, C_{size}\, C_{surface}\, C_{temp.}\, C_{reliability}\, S_f' \quad \text{---- } (*)$$

$$S_e = C_{load}\, C_{size}\, C_{surface}\, C_{temp.}\, C_{reliability}\, S_e'$$

Creating Estimated S-N Diagrams

Eq. (*) provide information about the material's strength in the high cycle region of the S-N diagram. With similar information for the low cycle region, we can construct an S-N diagram for the particular material and application. The bandwidth of interest is the high cycle fatigue region from 10^3 to 10^6 cycles and beyond.

Let material strength at 10^3 cycles be called S_o

bending	$S_o = 0.9\,\sigma_{ut}$
axial loading	$S_o = 0.75\,\sigma_{ut}$

If the material exhibit knee (ferrous material), then the corrected S_e from Eq. (*) is plotted at $N = 10^6$ cycles and a straight line is drawn between S_o and S_e. If the material does not exhibit a knee (non-ferrous materials) then the corrected S_f from Eq. (*) is plotted at the $N = 5\times10^8$ cycles and a straight line is drawn between S_o and S_f.

The eq. of line from S_o to S_e or S_f can be written as

$S_n = a N^b$ { used for ferrous materials if $N < 10^6$ cycles; for non-ferrous materials if $N < 5\times10^8$ cycles }