Parallel-Axis Theorem:

$$I_{A\overline{A}} = \int y^2 dA$$

C: centroid of the area
& $B\overline{B}$:centroidal axis

$$B\overline{B} \,//\, A\overline{A}$$

$$I_{A\overline{A}} = \int (\overline{y} + d)^2 dA$$
$$I_{A\overline{A}} = \int (\overline{y})^2 dA + 2d\int \overline{y} dA + d^2 \int dA$$

$\int (\overline{y})^2 dA = I_{B\overline{B}}$ = moment of inertia about the centroidal axis

$\int$ dA = Area
$\int \overline{y} dA$ = 1st moment of area about $B\overline{B}$
$\int \overline{y} dA$ = zero as $B\overline{B}$ is the centroidal axis

$$I = \overline{I} + Ad^2$$

$\overline{I}$: about the centroidal axis
I: about any axis // to the centroidal axis with a distance **d**

Put **$k^2 A$** for **I**

& $\overline{I} = (\overline{k})^2 A$

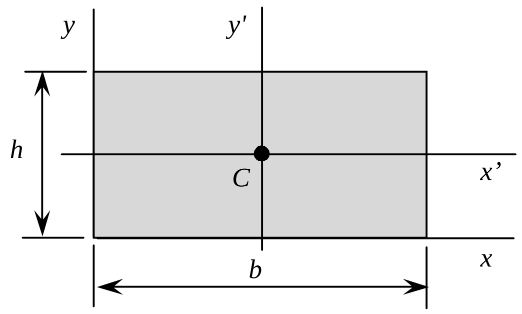
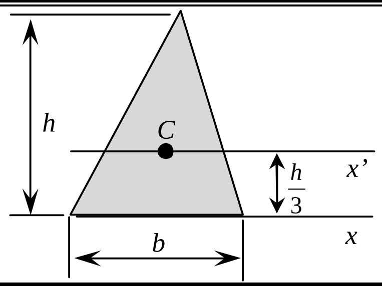
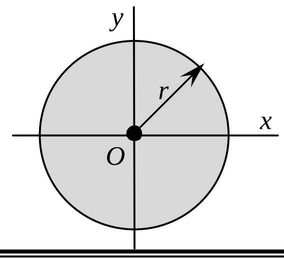
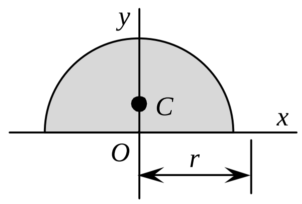
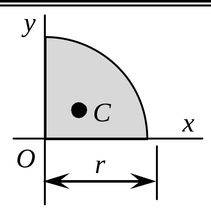
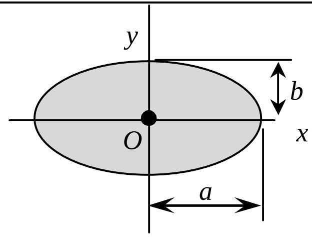
$$k^2 = (\overline{k})^2 + d^2$$

also

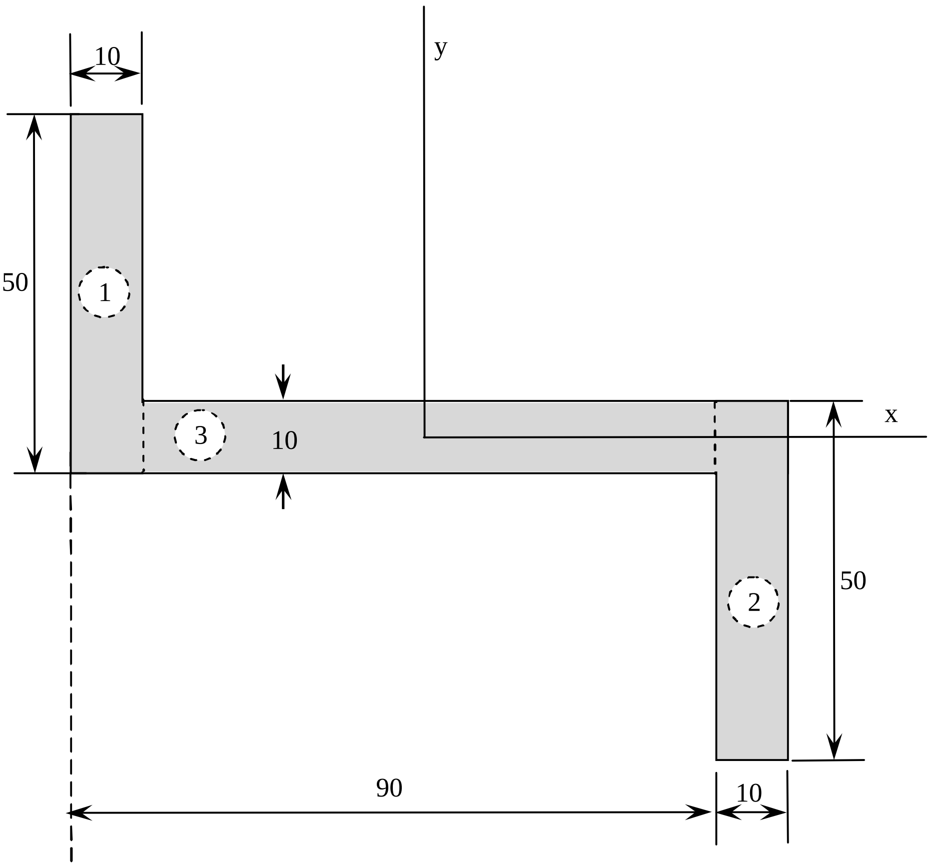
$$J_o = J_c + Ad^2$$

Moment of Inertia of Composite Area:

Moments of Inertia of common geometric shapes

Shape		Moments of inertia
Rectangle		$\bar{I}_{x'} = \frac{1}{12}bh^3$ $\bar{I}_{y'} = \frac{1}{12}b^3h$ $I_x = \frac{1}{3}bh^3$ $I_y = \frac{1}{3}b^3h$ $J_C = \frac{1}{12}bh(b^2 + h^2)$
Triangle		$\bar{I}_{x'} = \frac{1}{35}bh^3$ $I_x = \frac{1}{12}bh^3$
circle		$\bar{I}_x = \bar{I}_y = \frac{1}{4}\pi r^4$ $J_o = \frac{1}{2}\pi r^4$
Semicircle		$\bar{I}_x = \bar{I}_y = \frac{1}{8}\pi r^4$ $J_o = \frac{1}{4}\pi r^4$
Quarter Circle		$\bar{I}_x = \bar{I}_y = \frac{1}{16}\pi r^4$ $J_o = \frac{1}{8}\pi r^4$
Ellipse		$\bar{I}_x = \frac{1}{4}\pi ab^3$ $\bar{I}_y = \frac{1}{4}\pi a^3b$ $J_O = \frac{1}{4}\pi ab(a^2 + b^2)$

Ex: Find I_{xx} & k_x



$$\bar{I}_1 = \frac{b_1 h_1^3}{12} = \frac{10 * 50^3}{12}$$

$$\bar{I}_3 = \frac{b_3 h_3^3}{12} = \frac{80 * 10^3}{12}$$

$$I_{xx} = \bar{I}_3 + 2\left[\bar{I}_1 + Ad^2\right]$$

$$I_{xx} = \frac{80 * 10^3}{12} + 2\left[\frac{10 * 50^3}{12} + (50 * 10)(25 - 5)^2\right]$$

$$I_{xx} = 61.5 * 10^4\ mm^4$$

Or:

$$Part\ (1): I_x = \bar{I}_x + Ad^2 = \frac{b_1 h_1^3}{12} + Ad^2 = \frac{10 * 40^3}{12} + (40 * 10)(20 + 5)^2 = 303333.333\ mm^4$$

$$Part\ (2): I_x = \bar{I}_x + Ad^2 = \frac{b_2 h_2^3}{12} + Ad^2 = \frac{10 * 40^3}{12} + (40 * 10)(20 + 5)^2 = 303333.333\ mm^4$$

$$Part\ (3): I_x = \bar{I}_x + Ad^2 = \frac{b_3 h_3^3}{12} = \frac{100 * 10^3}{12} = 8333.333\ mm^4$$

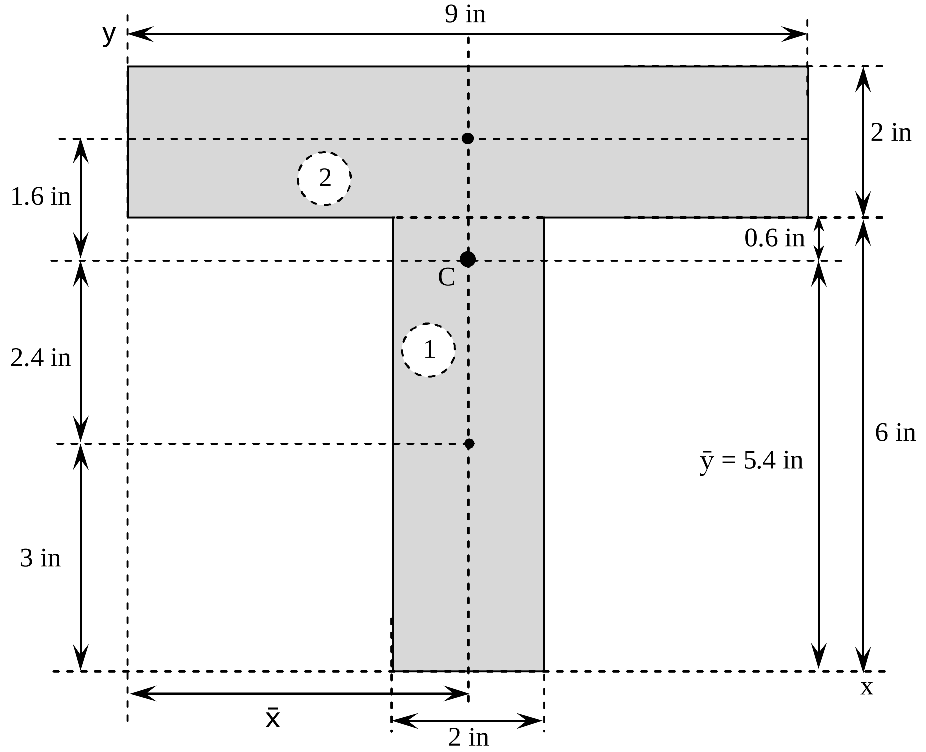
$$I_{xx} = \Sigma I_{xx} = I_1 + I_2 + I_3 = 303333.333 + 303333.333 + 8333.333 = 61.5 mm^4$$

$$A = 50 * 10 + 50 * 10 + 80 * 10 = 1800\ mm^2$$

$$k_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{61.5 * 10^4}{1800}} = 18.484\ mm$$

9.28

Determine the centroidal polar moment of inertia of the area shown.



Component	A	$\overline{x}$	$\overline{y}$	A $\overline{x}$	A $\overline{y}$
Rectangle 1	2 * 6 = 12	4.5	3	54	36
Rectangle 2	9 * 2= 18	4.5	7	81	126
	Σ A = 30			Σ A $\overline{x}$ = 135	Σ A $\overline{y}$ = 162

$$\overline{X} = \frac{\sum \overline{x}A}{A} = \frac{135}{30} = 4.5in$$

$$\overline{Y} = \frac{\sum \overline{y}A}{A} = \frac{162}{30} = 5.4in$$

$$Part\ (1): I_{x_1'} = \overline{I}_x + Ad^2 = \frac{b_1h_1^3}{12} + Ad^2 = \frac{2*(6)^3}{12} + (2*6)(2.4)^2 = 36 + 69.12$$

$$I_{x_1'} = 105.12\,in^4$$

$$Part\ (2): I_{x_2'} = \overline{I}_x + Ad^2 = \frac{b_2h_2^3}{12} + Ad^2 = \frac{9*(2)^3}{12} + (2*9)(1.6)^2 = 6 + 46.08$$

$$I_{x_1'} = 52.08\,in^4$$

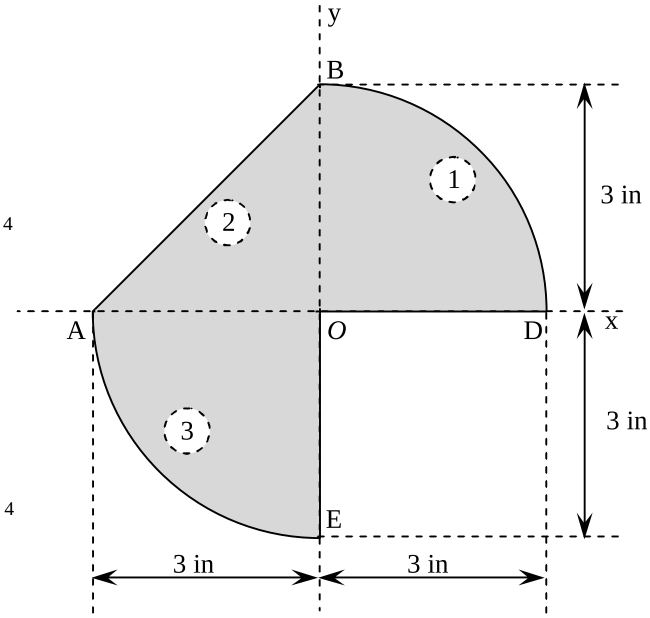
$$I_{x'} = I_{x_1'} + I_{x_2'} = 105.12 + 52.08 = 157.2in^4$$

$$I_{y'} = I_{y_1} + I_{y_2} = \frac{b_1^3h_1}{12} + \frac{b_2^3h}{12} = \frac{(2)^3*6}{12} + \frac{(9)^3*2}{12} = 4 + 121.5 = 125.5in^4$$

$$J_c = I_{x'} + I_{y'} = 157.2 + 125.5 = 282.7in^4$$

9.30

Determine the polar moment of inertia of the area shown with respect to (*a*) point O , (*b*) the centroid of the area .



(*a*)

$$I_x = I_{x_1} + I_{x_2} + I_{x_3}$$

$$I_x = \frac{1}{16}\pi r^4 + \frac{1}{12}bh^3 + \frac{1}{16}\pi r^4$$

$$I_x = \frac{1}{16}\pi * (3)^4 + \frac{1}{12} * 3 * (3)^3 + \frac{1}{16}\pi * (3)^4$$

$$I_x = 38.5586 \ in^4$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3}$$

$$I_y = \frac{1}{16}\pi r^4 + \frac{1}{12}b^3h + \frac{1}{16}\pi r^4$$

$$I_y = \frac{1}{16}\pi * (3)^4 + \frac{1}{12} * (3)^3 * 3 + \frac{1}{16}\pi * (3)^4$$

$$I_y = 38.5586 \ in^4$$

$$J_o = I_x + I_y = 38.5586 + 38.5586 = 77.11 in^4$$

(*b*)

$$Part\ (1): \bar{x} = \frac{+4r}{3\pi} = \frac{+4*3}{3\pi} = +1.273\ in.; \bar{y} = \frac{+4r}{3\pi} = \frac{+4*3}{3\pi} = +1.273\ in$$

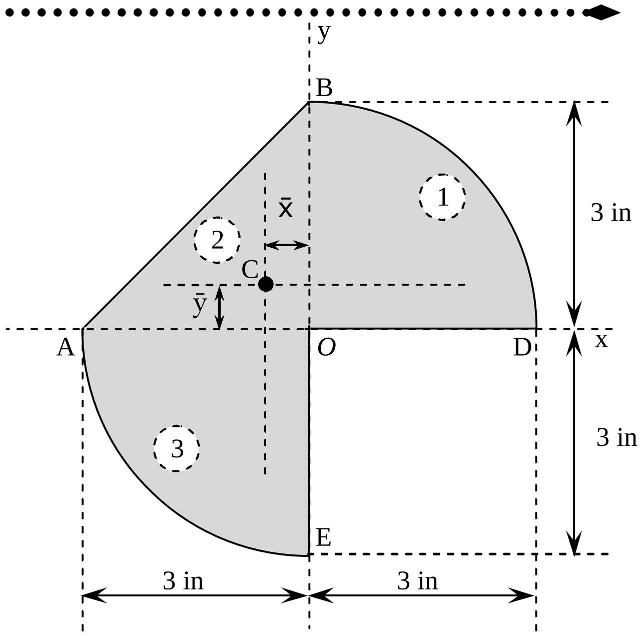
$$Part\ (2): \bar{x} = -\frac{b}{3} = \frac{-3}{3} = -1 in; \bar{y} = \frac{h}{3} = \frac{3}{3} = +1 in$$

$$Part\ (3): \bar{x} = \frac{-4r}{3\pi} = \frac{-4*3}{3\pi} = -1.273\ in.; \bar{y} = \frac{-4r}{3\pi} = \frac{-4*3}{3\pi} = -1.273\ in$$

Component	A	$\bar{x}$	$\bar{y}$	A $\bar{x}$	A $\bar{y}$
Quarter circle 1	$= \frac{\pi r^2}{4} = \frac{\pi * 3^2}{4}$ $= 7.068$	$= \frac{4r}{3\pi} = \frac{4*3}{3\pi}$ $= 1.2732$	$= \frac{4r}{3\pi} = \frac{4*3}{3\pi}$ $= 1.2732$	8.999	8.999
triangle 2	$= \frac{bh}{2} = \frac{3*3}{2}$ $= 4.5$	$= \frac{-b}{3} = \frac{-3}{3}$ $= -1$	$= \frac{h}{3} = \frac{3}{3}$ $= 1$	-4.5	4.5
Quarter circle 3	$= \frac{\pi r^2}{4} = \frac{\pi * 3^2}{4}$ $= 7.068$	$= \frac{-4r}{3\pi} = \frac{-4*3}{3\pi}$ $= -1.2732$	$= \frac{-4r}{3\pi} = \frac{-4*3}{3\pi}$ $= -1.2732$	- 8.999	- 8.999
----	Σ A = 18.636	----	----	Σ A $\bar{x}$ = -4.5	Σ A $\bar{y}$ = 4.5

$$\bar{x} = \frac{\sum \bar{x}A}{A} = \frac{-4.5}{18.636} = -0.242468 \ in$$

$$\bar{y} = \frac{\sum \bar{y}A}{A} = \frac{4.5}{18.636} = +0.241468 \ in$$



$$I_{x_1} = I_{x_1'} + A_1(\overline{y_1})^2$$

$$I_{x_1'} = \frac{1}{16}\pi r^4 - \frac{\pi r^2}{4} * (1.273)^2$$

$$I_{x_1'} = \frac{1}{16}\pi * 3^4 - \frac{\pi\, 3^2}{4} * (1.273)^2$$

$$I_{x_1'} = 4.45\, in^4$$

$$I_{cx_1} = I_{x_1'} + A_1(\overline{y_1} - \overline{y})^2$$

$$I_{cx_1} = 4.45 + \frac{\pi r^2}{4} * (1.273 - 0.2414)^2$$

$$I_{cx_1} = 4.45 + \frac{\pi\, 3^2}{4} * (1.0316)^2$$

$$I_{cx_1} = 11.9723\ in^4$$

$$I_{x_2} = I_{x_2'} + A_2(\overline{y_2})^2$$

$$I_{x_2'} = \frac{1}{12} bh^3 - \frac{bh}{2} * (1)^2 = \frac{1 * 3 * 3^3}{12} - \frac{3 * 3}{2} * (1)^2$$

$$I_{x_2'} = 2.25\, in^4$$

$$I_{cx_2} = I_{x_2'} + A_2(\overline{y_2} - \overline{y})^2$$

$$I_{cx_2} = 2.25 + \frac{bh}{2} * (1 - 0.2414)^2 = 2.25 + \frac{3 * 3}{2} * (0.7586)^2$$

$$I_{cx_2} = 4.8396\ in^4$$

$$I_{x_3} = I_{x_3'} + A_3(\overline{y_3})^2$$

$$I_{x_3'} = \frac{1}{16}\pi r^4 - \frac{\pi r^2}{4} * (1.273)^2 = \frac{1}{16}\pi * 3^4 - \frac{\pi\, 3^2}{4} * (1.273)^2$$

$$I_{x_3'} = 4.45\, in^4$$

$$I_{cx_3} = I_{x_3'} + A_3(\overline{y_3} + \overline{y})^2$$

$$I_{cx_3} = 4.45 + \frac{\pi r^2}{4} * (1.273 + 0.2414)^2 = 4.45 + \frac{\pi\, 3^2}{4} * (1.5144)^2$$

$$I_{cx_3} = 20.6611\ in^4$$

$$I_{cx} = I_{cx_1} + I_{cx_2} + I_{cx_3}$$

$$I_{cx} = 11.9723 + 4.839 + 20.6611$$

$$I_{cx} = 37.473\ in^4$$

$$I_{y_1} = I_{y_1'} + A_1(\overline{x_1})^2$$

$$I_{y_1'} = \frac{1}{16}\pi r^4 - \frac{\pi r^2}{4} * (1.273)^2 = \frac{1}{16}\pi * 3^4 - \frac{\pi\, 3^2}{4} * (1.273)^2$$

$$I_{y_1'} = 4.45\, in^4$$

$$I_{cy_1} = I_{y_1'} + A_1(\overline{x_1} + \overline{x})^2$$

$$I_{cy_1} = 4.45 + \frac{\pi r^2}{4} * (1.273 + 0.2414)^2 = 4.45 + \frac{\pi 3^2}{4} * (1.5144)^2$$

$$I_{cy_1} = 20.6611\ in^4$$

$$I_{y_2} = I_{y_2'} + A_2(\overline{x_2})^2$$

$$I_{y_2'} = \frac{1}{12} b^3 h - \frac{bh}{2} * (1)^2 = \frac{1 * 3^3 * 3}{12} - \frac{3 * 3}{2} * (1)^2$$

$$I_{y_2'} = 2.25\ in^4$$

$$I_{cy_2} = I_{y_2'} + A_2(\overline{x_2} - \overline{x})^2$$

$$I_{cy_2} = 2.25 + \frac{bh}{2} * (1 - 0.2414)^2 = 2.25 + \frac{3 * 3}{2} * (0.7586)^2$$

$$I_{cy_2} = 4.8396\ in^4$$

$$I_{y_3} = I_{y_3'} + A_3(\overline{x_3})^2$$

$$I_{y_3'} = \frac{1}{16} \pi r^4 - \frac{\pi r^2}{4} * (1.273)^2 = \frac{1}{16} \pi * 3^4 - \frac{\pi 3^2}{4} * (1.273)^2$$

$$I_{y_3'} = 4.45\ in^4$$

$$I_{cy_3} = I_{xy_3'} + A_3(\overline{x_3} - \overline{x})^2$$

$$I_{cy_3} = 4.45 + \frac{\pi r^2}{4} * (1.273 - 0.2414)^2 = 4.45 + \frac{\pi 3^2}{4} * (1.0316)^2$$

$$I_{cy_3} = 11.9723\ in^4$$

$$I_{cy} = I_{cy_1} + I_{cy_2} + I_{cy_3}$$

$$I_{cx} = 20.6611 + 4.839 + 11.9723$$

$$I_{cx} = 37.473\ in^4$$

$$J_c = I_{cx} + I_{cy}$$

$$J_c = 37.473 + 37.573$$

$$J_c = 74.946\ in^4$$