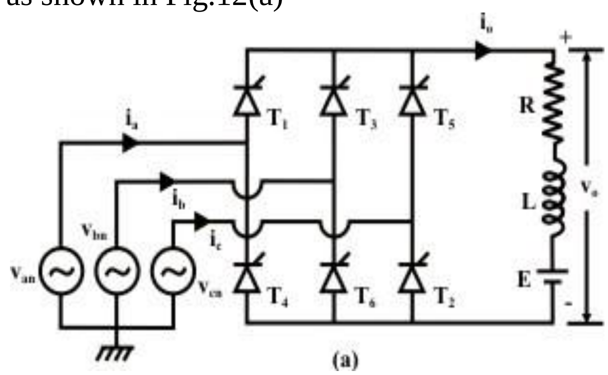
Lesson 4
Operation and Analysis of the Three Phase Fully Controlled Bridge Converter

6. Operating principle of 3 phase fully controlled bridge converter

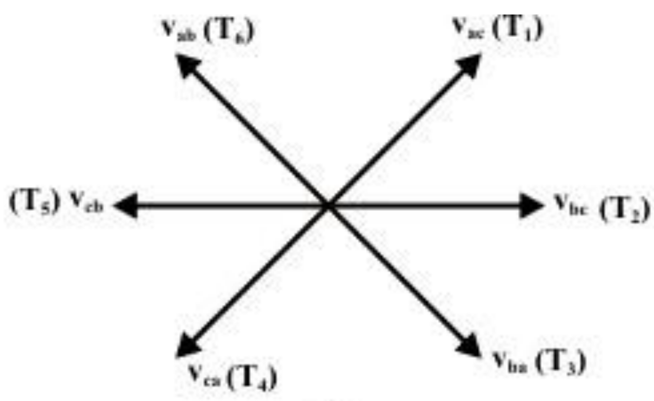
A three phase fully controlled converter is obtained by replacing all the six diodes of an uncontrolled converter by six thyristors as shown in Fig.12(a)

Device / Mode	v_{T1}	v_{T2}	v_{T3}	v_{T4}	v_{T5}	v_{T6}	v_o
T_1T_2	0	0	v_{ba}	v_{ca}	v_{ca}	v_{cb}	v_{ac}
T_2T_3	v_{ab}	0	0	v_{ca}	v_{cb}	v_{cb}	v_{bc}
T_3T_4	v_{ab}	v_{ac}	0	0	v_{cb}	v_{ab}	v_{ba}
T_4T_5	v_{ac}	v_{ac}	v_{bc}	0	0	v_{ab}	v_{ca}
T_5T_6	v_{ac}	v_{bc}	v_{bc}	v_{ba}	0	0	v_{cb}
T_6T_1	0	v_{bc}	v_{ba}	v_{ba}	v_{ca}	0	v_{ab}
NONE	-	-	-	-	-	-	E



(a)

(b)



(c)

Fig.12; Operation of a three phase full controlled bridge converter
- **(a) Circuit diagram**
- **(b) Conduction table**
- **(c) Phase diagram of line voltages**

For any current to flow in the load at least one device from the top group (T_1, T_3, T_5) and one from the bottom group (T_2, T_4, T_6) must conduct. It can be argued as in the case of an uncontrolled converter only one device from these two groups will conduct.

Then from symmetry consideration it can be argued that each thyristor conducts for 120° of the input cycle. Now the thyristors are fired in the sequence $T_1 \rightarrow T_2 \rightarrow T_3 \rightarrow T_4 \rightarrow T_5 \rightarrow T_6 \rightarrow T_1$ with 60° interval between each firing. Therefore thyristors on the same phase leg are fired at an interval of 180° and hence can not conduct simultaneously. This leaves only six possible conduction mode for the converter in the continuous conduction mode of operation. These are T_1T_2, T_2T_3, T_3T_4, T_4T_5, T_5T_6, T_6T_1. Each conduction mode is of 60° duration and appears in the sequence mentioned. The conduction table of Fig.12 (b) shows voltage across different devices and the dc output voltage for each conduction interval. The phasor diagram of the line voltages appear in Fig. 12(c). Each of these line voltages can be associated with the firing of a thyristor with the help of the conduction table-1. For example the thyristor T_1 is fired at the end of T_5T_6 conduction interval. During this period the voltage across T_1 was v_{ac}. Therefore T_1 is fired α angle after the positive going zero crossing of v_{ac}. Similar observation can be made about other thyristors. The phasor diagram of Fig. 12 (c) also confirms that all the thyristors are fired in the correct sequence with 60° interval between each firing.

Fig.12 shows the waveforms of different variables (shown in Fig.12 (a)). To arrive at the waveforms it is necessary to draw the conduction diagram which shows the interval of conduction for each thyristor and can be drawn with the help of the phasor diagram of fig. 12(c). If the converter firing angle is α each thyristor is fired "α" angle after the positive going zero crossing of the line voltage with which it's firing is associated. Once the conduction diagram is drawn all other voltage waveforms can be drawn from the line voltage waveforms and from the conduction table of fig.12 (b). Similarly line currents can be drawn from the output current and the conduction diagram. It is clear from the waveforms that output voltage and current waveforms are periodic over one sixth of the input cycle. Therefore this converter is also called the "six pulse" converter. The input current on the other hand contains only odds harmonics of the input frequency other than the triplex (3^{rd}, 9^{th} etc.) harmonics. The next section will analyze the operation of this converter in more details.

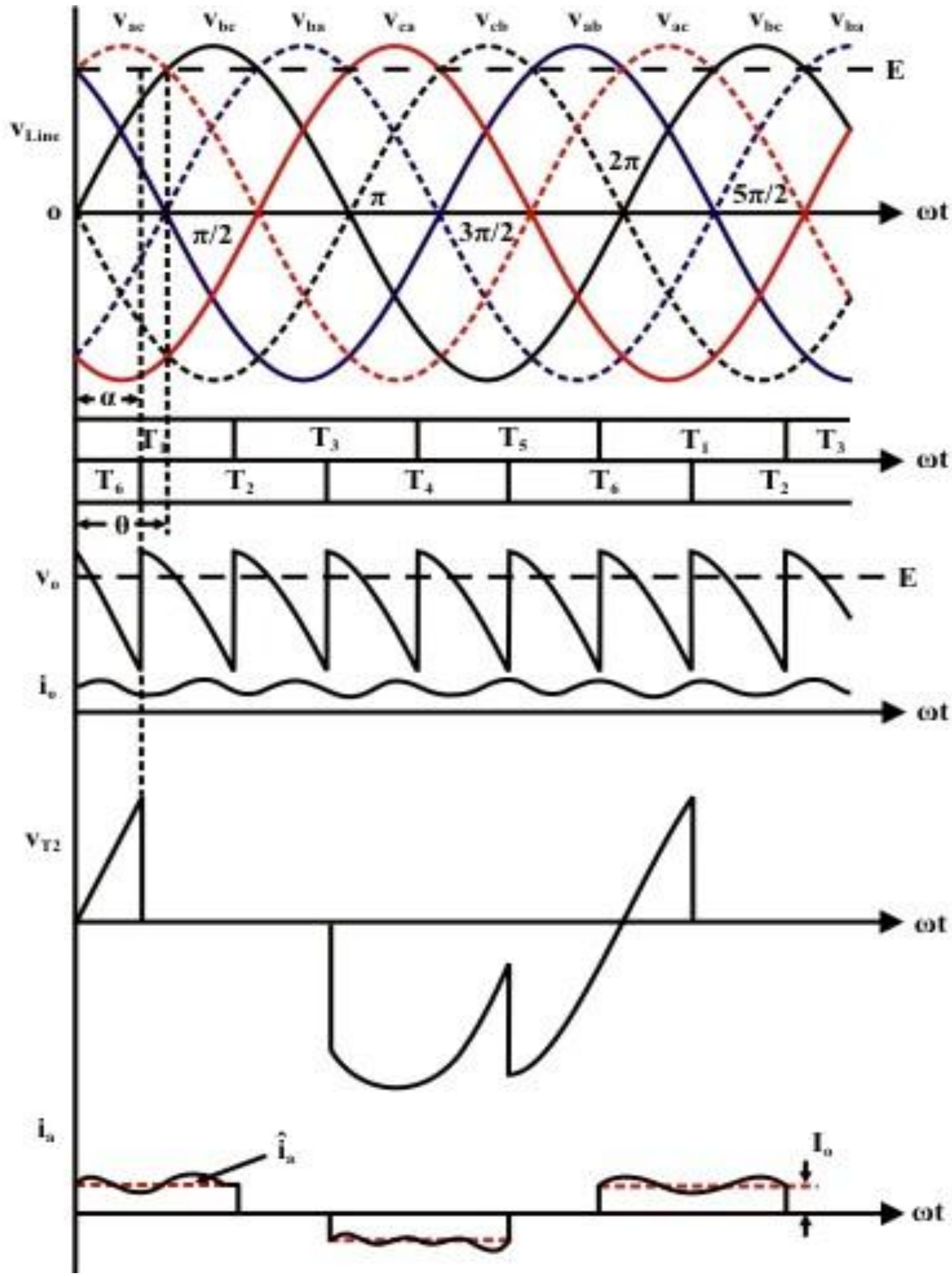


Fig.13; Waveforms of three phase full controlled bridge converter

Exercise 6

Fill in the blank(s) with the appropriate word(s)

i) The three phase fully controlled bridge converter is obtained by replacing six _________ of an uncontrolled converter by six __________.

ii) The pulse number of a three phase fully controlled bridge converter is _________.

iii) In a three phase fully controlled converter each device conducts for an interval of ___________ degrees.

iv) In a three phase fully controlled converter operating in continuous conduction there are _________ different conduction modes.

v) The output voltage of a three phase fully controlled converter operating in the continuous conduction mode consists of segments of the input ac _________ voltage.

vi) The peak voltage appearing across any device of a three phase fully controlled converter is equal to the _________ input ac _________ voltage.

vii) The input ac current of a three phase fully controlled converter has a _________ step waveform.

viii) The input ac current of a three phase fully controlled converter contains only __________ harmonics but no __________ harmonic.

ix) A three phase fully controlled converter can also operate in the __________ mode.

x) Discontinuous conduction in a three phase fully controlled converter is _________.

Answers: (i) diodes, thyristors; (ii) six; (iii) 120; (iv) six; (v) line; (vi) peak, line; (vii) six; (viii) odd, tripler; (ix) inverting; (x) rare.

7. Analysis of the converter in the rectifier mode

The output voltage waveform can be written as

$$v_0 = V_0 + \sum_{K=1,2}^{\alpha} V_{AK} \cos 6\,K\omega t + \sum_{K=1,2}^{\alpha} V_{BK} \sin 6\,K\omega t \tag{40}$$

$$V_0 = \frac{3}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \, d\omega t = \frac{3\sqrt{2}}{\pi} V_L \int_{\alpha}^{\alpha+\frac{\pi}{3}} \sin\left(\omega t + \frac{\pi}{3}\right) d\omega t$$

$$= \frac{3\sqrt{2}}{\pi} V_L \cos\alpha \tag{41}$$

$$V_{AK} = \frac{6}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \cos 6\,K\omega t \, d\omega t$$

$$= \frac{6}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} \sqrt{2}\, V_L \sin\left(\omega t + \frac{\pi}{3}\right) \cos 6\,\omega t \, d\omega t$$

$$= \frac{3\sqrt{2}}{\pi} V_L \left[\frac{\cos(6K+1)\alpha}{6K+1} - \frac{\cos(6K-1)\alpha}{6K-1}\right] \tag{42}$$

$$V_{BK} = \frac{6}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0 \sin 6\,K\omega t \, d\omega t$$

$$= \frac{6}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} \sqrt{2}\, V_L \sin\left(\omega t + \frac{\pi}{3}\right) \sin 6\,\omega t \, d\omega t$$

$$= \frac{3\sqrt{2}}{\pi} V_L \left[\frac{\sin(6K+1)\alpha}{6K+1} - \frac{\sin(6K-1)\alpha}{6K-1}\right] \tag{43}$$

$$V_{0RMS} = \sqrt{\frac{3}{\pi}\int_{\alpha}^{\alpha+\frac{\pi}{3}} v_0^2 \, d\omega t} = V_L \left[1 + \frac{3\sqrt{3}}{4\pi}\cos 2\alpha\right]^{\frac{1}{2}}$$

The input phase current i_a is expressed as

$$i_a = i_0 \qquad \alpha \le \omega t \le \alpha + \frac{\pi}{3}$$

$$i_a = -i_0 \qquad \alpha + \frac{2\pi}{3} \le \omega t \le \alpha + \frac{4\pi}{3}$$

$$i_a = i_0 \qquad \alpha + \frac{5\pi}{3} \le \omega t \le \alpha + 2\pi$$

$$i_a = 0 \qquad \text{otherwise}$$

From Fig.13 it can be observed that i_0 itself has a ripple at a frequency six times the input frequency. The closed from expression of i_0, as will be seen later is some what complicated. However, considerable simplification in the expression of i_a can be obtained if i_0 is replaced by its average value I_0. This approximation will be valid provided the ripple on i_0 is small, i.e, the load is highly inductive. The modified input current waveform will then be i_a which can be expressed in terms of a fourier series as

$$i_a \approx \hat{i}_a = \frac{I_{A0}}{2} + \sum_{n=1}^{\alpha} I_{An} \cos n\omega t + \sum_{n=1}^{\alpha} I_{Bn} \sin n\omega t \qquad (44)$$

Where

$$I_{A0} = \frac{1}{2\pi}\int_{\alpha}^{\alpha+2\pi} i_a \, d\omega t = 0 \qquad (13.6)$$

$$I_{An} = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} i_a \cos n\omega t \qquad n \neq 0$$

$$= \frac{4I_0}{n\pi}\cos\frac{n\pi}{6}\sin\frac{n\pi}{2}\cos n\alpha \qquad (45)$$

$$\therefore \qquad I_{An} = (-1)^K \frac{2\sqrt{3}I_0}{(6K \pm 1)\pi}\sin\left(K\pi \pm \frac{\pi}{2}\right)\cos(6K \pm 1)\alpha$$

$$\text{for } n = 6K \pm 1, K = 0, 1, 2, 3 \ldots \qquad (46)$$

$I_{An} = 0$ otherwise.

$$I_{Bn} = \frac{1}{\pi}\int_{\alpha}^{\alpha+2\pi} i_a \sin n\omega t \, d\omega t$$

$$= \frac{4I_0}{n\pi}\cos\frac{n\pi}{6}\sin n\alpha \sin\frac{n\pi}{2} \qquad (47)$$

$$\therefore \qquad I_{Bn} = (-1)^K \frac{2\sqrt{3}I_0}{(6K \pm 1)\pi}\sin\left(K\pi \pm \frac{\pi}{2}\right)\sin(6K \pm 1)\alpha$$

$$\text{for } n = 6K \pm 1, K = 0, 1, 2, \ldots \qquad (48)$$

$I_{Bn} = 0$ otherwise.

$$\therefore \qquad i_a = \sum_{K=0}^{\alpha} \frac{(-1)^K 2\sqrt{3}I_0}{(6K \pm 1)\pi}\sin\left(K\pi \pm \frac{\pi}{2}\right)\cos\left[(6K \pm 1)(\omega t - \alpha)\right] \qquad (49)$$

in particular i_{a1} = fundamental component of i_a

$$= \frac{2\sqrt{3}}{\pi} I_0 \cos(\omega t - \alpha) \qquad (50)$$

From Fig. 13.2 $v_{an} = \frac{\sqrt{2}V_L}{\sqrt{3}} \cos \omega t$ (51)

$\therefore$ displacement angle φ = α. $\therefore$ displacement factor = cosα (52)

distortion factor = $\frac{I_{a1}}{I_a} = \left(\frac{\sqrt{6}}{\pi}\right)^{I_0} \Big/ \sqrt{\frac{2}{3}} I_0 = \frac{3}{\pi}$ (53)

$\therefore$ Power factor = Displacement factor × Distortion factor = $\frac{3}{\pi}\cos\alpha$ (54)

The closed form expression for i_0 in the interval $\alpha \le \omega t \le \alpha + \frac{\pi}{3}$ can be found as follows

in this interval

$$Ri_0 + L\frac{di_0}{dt} + E = v_0 = \sqrt{2}V_L \sin\left(\omega t + \frac{\pi}{3}\right) \quad (55)$$

$$i_0 = I_1 e^{-\frac{(\omega t - \alpha)}{\tan\varphi}} + \frac{\sqrt{2}V_L}{Z}\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{E}{R} \quad (56)$$

Where $Z = \sqrt{R^2 + \omega^2 L^2}, \quad \tan\varphi = \frac{\omega L}{R}$

$\therefore \quad R = Z\cos\varphi, \; E = \sqrt{2}V_L \sin\theta$ (from Fig. 13.2) (57)

$$\therefore \quad i_0 = I_1 e^{-\frac{(\omega t - \alpha)}{\tan\varphi}} + \frac{\sqrt{2}V_L}{Z}\left[\sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right] \quad (58)$$

Since i_0 is periodic over π/3

$$i_0\big|_{\omega t = \alpha} = i_0\big|_{\omega t = \alpha + \frac{\pi}{3}} \quad (59)$$

$$\therefore \quad I_1 + \frac{\sqrt{2}V_L}{Z}\left[\sin\left(\alpha + \frac{\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right]$$

$$= I_1 e^{-\frac{\pi}{3\tan\varphi}} + \frac{\sqrt{2}V_L}{Z}\left[\sin\left(\alpha + \frac{2\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right]$$

OR $$I_1 = \frac{\sqrt{2}V_L}{Z}\ \frac{\sin(\varphi - \alpha)}{1 - e^{-\frac{\pi}{3\tan\varphi}}} \qquad (60)$$

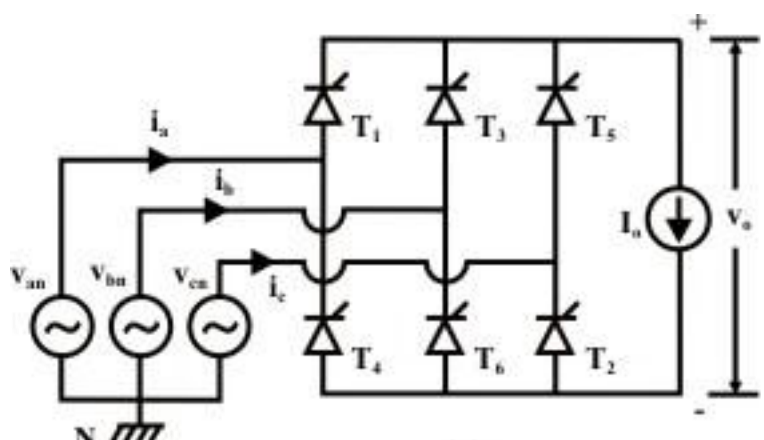
$$\therefore \qquad i_0 = \frac{\sqrt{2}V_L}{Z}\left[\frac{\sin(\varphi - \alpha)}{1 - e^{-\frac{\pi}{3\tan\varphi}}} e^{-\frac{(\omega t - \alpha)}{\tan\varphi}} + \sin\left(\omega t + \frac{\pi}{3} - \varphi\right) - \frac{\sin\theta}{\cos\varphi}\right] \qquad (61)$$

To find out the condition for continuous conduction it is noted that in the limiting case of continuous conduction.

$i_0\big|_{min=0}$, Now if $\theta \le \alpha + \frac{\pi}{3}$ then i_0 is minimum at $\omega t = \alpha$. $\therefore$ Condition for continuous conduction is $i_0\big|_{\omega t=\alpha} \ge 0$. However discontinuous conduction is rare in these conversions and will not be discussed any further.

8. Analysis of the converter in the inverting mode.

In all the analysis presented so far it has been assumed that $\alpha < 90°$. It follows from equation 13.2 that the output dc voltage will be positive in this case and power will be flowing from the three phase ac side to the dc side. This is the rectifier mode of operation of the converter. However if α is made larger than 90° the direction of power flow through the converter will reverse provided there exists a power source in the dc side of suitable polarity. The converter in that case is said to be operating in the inverter mode. It has been explained in connection with single phase converters that the polarity of EMF source on the dc side [Fig. 12(a)] would have to be reversed for inverter mode of operator. Fig. 14 shows the circuit connection and wave forms in the inverting mode of operation where the load current has been assumed to be continuous and ripple free.



(a)

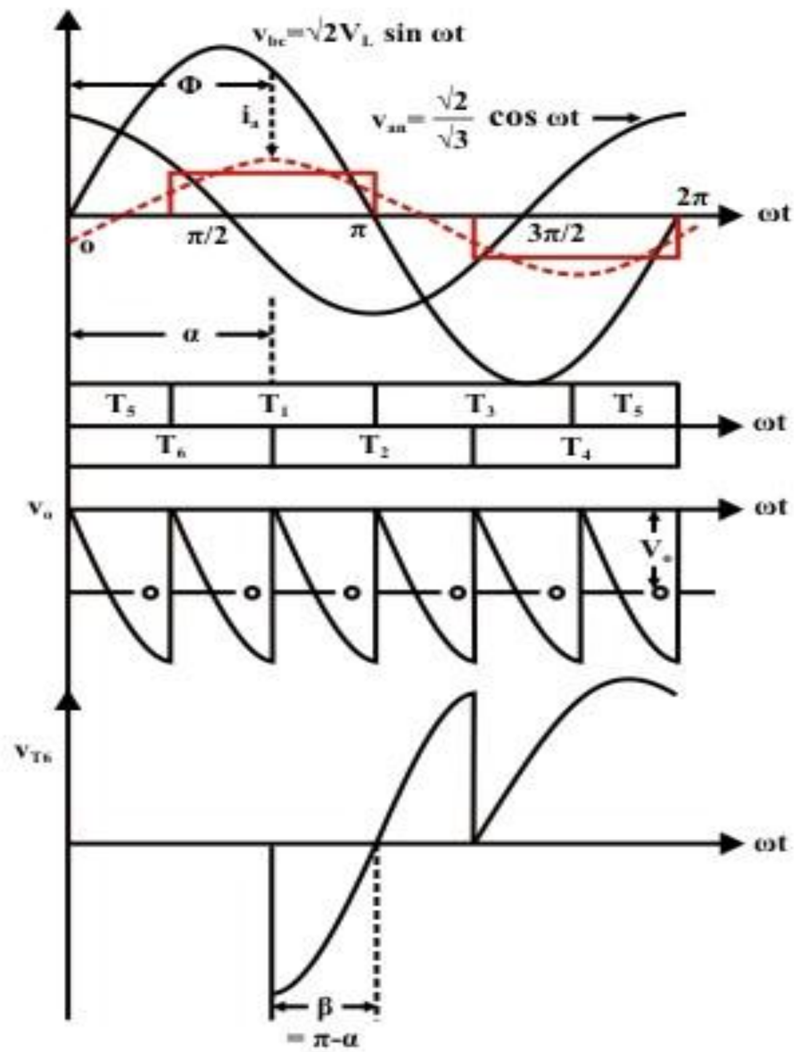


Fig.14: Inversion mode of operation of three phase full controlled bridge converter
(a) circuit diagram
(b) waveforms

Analysis of the converter in the inverting mode is similar to its rectifier mode of operation. The same expressions hold for the dc and harmonic compounds in the output voltage and current. The input supply current Fourier series is also identical to Equation 48. In particular

$$V_0 = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha \qquad (62)$$

$$i_{a1} = \frac{2\sqrt{3}}{\pi} I_0 \cos(\omega t - \alpha) \qquad (63)$$

For values of α in the range 90° < α < 180° it is observed from Fig.14 (b) that the average dc voltage is negative and the displacement angle φ of the fundamental component of the input ac line current is equal to α > 90°. Therefore, power in the ac side flows from the converter to the source.

It is observed form Fig. 14(b) that an outgoing thyristor (thyristor T_6 in Fig. 14(b)) after commutation is impressed with a negative voltage of duration $\beta = \pi - \alpha$. For successful commutation of the outgoing thyristor it is essential that this interval is larger than the turn off time of the thyristor i.e,

$\beta \geq \omega tq$, tq is the thyristor turn off time

Therefore $\pi - \alpha \geq \omega tq$ or $\alpha \leq \pi - \omega tq$ . .

Which imposes an upper limit on the value of α. In practice this upper value of α is further reduced due to commutation overlap.

Exercise 7

1. A three phase fully controlled bridge converter operating from a 3 phase 220 V, 50 Hz supply is used to charge a battery bank with nominal voltage of 240 V. The battery bank has an internal resistance of 0.01 Ω and the battery bank voltage varies by ± 10% around its nominal value between fully charged and uncharged condition. Assuming continuous conduction find out.

(i) The range of firing angle of the converter.
(ii) The range of ac input power factor.
(iii) The range of charging efficiency.

When the battery bank is charged with a constant average charging current of 100 Amps through a 250 mH lossless inductor.

Answer: The maximum and minimum battery voltages are, $V_{B\ Min} = 0.9 \times V_{B\ Nom} =$ 216 volts and $V_{B\ Max} = 1.1 \times V_{B\ Nom} = 264$ volts respectively.

Since the average charging current is constant at 100 A.

$V_{0\ Max} = V_{B\ Max} + 100 \times R_B = 264 + 100 \times 0.01 = 265$ volts
$V_{0\ Min} = V_{B\ Min} + 100 \times R_B = 216 + 100 \times 0.01 = 217$ volts.

(i) But $V_{0\ Max} = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha_{Min}$ $\therefore \alpha_{Min} = 26.88^o$

$V_{0\ Min} = \frac{3\sqrt{2}}{\pi} V_L \cos\alpha_{Max}$ $\therefore \alpha_{Max} = 43.08^o$

(ii) Input power factor is maximum at minimum α and vice versa

∴

$$\text{p.f. max} = \text{Distortion factor} \times \text{Displacement factor}\Big|_{Max} = \frac{3}{\pi}\times\cos\alpha_{min} = 0.85$$

$$\text{p.f. Min} = \frac{3}{\pi}\times\cos\alpha_{Max} = 0.697$$

(iii) Power loss during charging = $I_{0RMs}^2 R_B$

But $I_{0RMs}^2 = I_0^2 + I_1^2 + I_2^2 + \ldots\ldots..$ and $I_K \approx \frac{V_K}{6K\omega L} = \frac{\sqrt{V_{AK}^2 + V_{BK}^2}}{6\sqrt{2}K\omega L}$

For $\alpha = \alpha_{Min}$

$V_{A1} = 0.439$ V, $V_{B1} = 48.48$ V, $I_1 = 0.073$ Amps

$V_{A2} = 10.76$, $V_{B2} = 20.15$ V, $I_2 = 0.017$ Amps.

∴ $J_{0RMs}^2 \approx 100^2 + (0.073)^2 + (0.017)^2 = 10000.00562$

∴ $P_{loss} = 100$ watts.

At α_{Min}, $P_0 = I_0 \times V_{B\,Max} = 26400$ watts.

∴ Charging efficiency = $\frac{26400}{26400 + 100} = 99.6\%$

Similarly for α_{Max}, $I_{0RMs}^2 \approx I_0^2$

∴ $P_{loss} = 100$ watts

$P_0 = I_0 \times V_{B\,Min} = 21600$ watts.

∴ Charging efficiency = $\frac{21600}{21600 + 100} = 99.54\%$

2. A three phase fully controlled converter operates from a 3 phase 230 V, 50 Hz supply through a Y/Δ transformer to supply a 220 V, 600 rpm, 500 A separately excited dc motor. The motor has an armature resistance of 0.02 Ω. What should

be the transformer turns ratio such that the converter produces rated motor terminal voltage at 0º firing angle. Assume continuous conduction. The same converter is now used to brake the motor regeneratively in the reverse direction. If the thyristors are to be provided with a minimum turn off time of 100 μs, what is the maximum reverse speed at which rated braking torque can be produced.

Answer: From the given question

$$\frac{3\sqrt{2}}{\pi} V_L = 220$$

$\therefore$ V_L = 162.9 V

Where V_L is the secondary side line and also the phase voltage since the secondary side is Δ connected.

Primary side phase voltage = $\frac{230}{\sqrt{3}} V$ = 132.79 V

$\therefore$ Turns ratio = $\frac{132.79}{162.9} = 1:1.2267$.

During regenerative braking in the reverse direction the converter operates in the inverting mode.

$$tq\Big|_{Min} = 100 \mu S \quad \therefore \quad \beta_{Min} = \omega tq\Big|_{Min} = 1.8^{\circ}$$

$\therefore$ $\alpha_{Max} = 180 - \beta_{Min} = 178.2^{\circ}$

$\therefore$ Maximum negative voltage that can be generated by the converter is

$$\frac{3\sqrt{2}}{\pi} V_L \cos 178.2^{\circ} = -219.89 \text{ V}$$

For rated braking torque I_a = 500 A

$\therefore$ $E_b = V_a - I_a r_a$ = - 229.89 V.

At 600 RPM E_b = 220 – 500 × 0.02 = 210 V.

$\therefore$ Max reverse speed is $\frac{229.89}{210} \times 600 = 656.83$ RPM.